STUDY OF ADAPTIVE MULTISAMPLING IN MATCHED FILTERS AND DECODERS

A Thesis Submitted
in Partial Fulfilment of the Requirements
for the Degree of

MASTER OF TECHNOLOGY

by

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to the

DEPARTMENT OF ELECTRICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY KANPUR
MAY, 1986

C E R T I F I C A T E

THIS is to certify that the thesis entitl 'STUDY OF ADAPTIVE MULTISAMPLING IN MATCHED FILTERS AND DECODERS' has been carried out by Shri Siddhartha Ray Chaudhuri (Roll No. 8410444) under my supervision and that this has not been submitted elsewhere for the award of any degree.

Place : Kanpur

Date : 05/05/86.

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D E D I C A T I O N

Dedicated to

my Parents,

my Teachers, and

the very many workers

in this field

A C K N O W L E D G E M E N T

IT is with great pleasure that I acknowledge the invaluable guidance, assistance and encouragement that I obtained from my thesis supervisor Prof. J. Das throughout the period of work. I wish to place on record my deep sense of appreciation for his wise counsel and his painstakingly elaborate explanation for even the silliest of my silliest questions. To me he has been much more than a thesis supervisor.

I also wish to place on record my grateful acknowledgement of the numerous help provided directly or indirectly on academic as well as nonacademic matters by Prof. P.K. Chatterjee, without whose constant encouragement the present work might not have taken shape. It was he who first gave me the basic hints and tricks to do and effectively use computer simulation, which finally turned out to be one of the basic pillars of the present work. My sincerest thanks are also due to Profs.S.K. Mullick and V. Sinha for having provided me with the useful study-aids from time to time. I am also indebted to Shri P.G. Poonacha for his help and suggestions throughout the entire period of the thesiswork.

I consider myself lucky when it comes to the company of friends that I have had and enjoyed during the entire period of my study at IIT Kanpur. It is difficult if not impossible to acknowledge their many visible as well as invisible help individually. But I feel I will be guilty of a cautious oblivion if I fail to acknowledge the ever-pleasing (!) and ever-encouraging (!!) company of my pals and seniors Soumyesh, Partha, Utpal, Sujoy, Ujjwal, Chandoo, Tarun, Dipayan, Sourav, Somnath (Bastu), Mrityunjoy, Subir, Maheshwar (Nakur), Gharghar, Debajyotida, Amarnathda, and others.

Finally I wish to convey my heartfelt gratitude to Mr. S.N. Sikdar. But for the timely help extended by him the thesis could not have been typed. It was he who made all the necessary arrangements so that the thesis could be typed into its presentable form within a very short period of time. Thanks are also due to Mr. Gupta of the E.E. Stores for having extended his cooperation in typing the thesis to its final shape.

Siddhartha Ray Chaudhuri

(AUTHOR)

ABSTRACT

THE topic of the thesis is 'Study of Adaptive Multisampling in Matched Filters and Decoders', and concerns the theoretical and experimental evaluation of the improvement in performance obtained with coded data reception when the technique of Multisampling is employed. Conventional receivers sample the code chips only at the respective optimum points and the samples are processed as if they are the representative of the chips that have been sampled. The technique of multisampling on the other hand proposes to obtain greater insight into the chips by sampling them multiply inside their respective signalling intervals. The work proposes methods for manipulating the multiple samples of a chip so that the overall error rate and SNR performances are better than those with single sampling. As a useful application, the SNR gain obtained in multisampling may be utilized to extract soft decision performance out of simpler hard decision receivers. In specific cases, therefore, the technique of multisampling may be able to replace conventional multibit quantization of the samples and thus result in a gross simplification of the receiver configuration without degrading the performance. In several other cases, it may coact with multibit quantization to result in receivers that are more cost effective than at present. Simulation results are used for experimental verification of the theory wherever necessary.

CONTENTS

CHAPTER 1: INTRODUCTION

TODAY communication enters the daily life in so many different ways that it is easy to overlook the multitude of its facets. In the most fundamental sense, communication involves implicitly the transmission of information from one point to another through a succession of processes as outlined below.

(a) The generation of a thought pattern in the mind of an originator;

(b) The description of that thought pattern within a certain measure of precision by a set of symbols;

(c) The encoding and modulation of these symbols in a form that is suitable for transmission over a medium of interest;

(d) The transmission of the encoded and modulated symbols and their reception at the desired destination;

(e) The demodulation, detection and decoding of the initial symbols; and, finally,

(f) The recreation of the original thought pattern in the mind of the recipient, with a definable degradation in quality of the said pattern. The degradations are due to the imperfections of the system.

As far as the processes in (c), (d), and (e) above are concerned, it has been a common trend to use digital systems as far as practicable. It is commonplace to hear people say the world is going digital. This is true not only for communications, but for many other systems starting from the small wristwatches till the mainframe computers. The reasons are not hard to think out and one can readily speak of the following advantages of a digital system over the analog counterpart .

(a) Less dependence on component tolerance;

(b) Highly consistent accuracy;

(c) Smaller physical size;

(d) Ease and economy of implementation;

(e) High reliability; and,

(f) Easy adaptability;

The field of digital communication has seen an enormous growth since the publication of C.E. Shannon's pioneering work in 1948 [11]. He demonstrated analytically that if the data source rate is less than a quantity called the Channel Capacity, communication over a noisy channel is possible with proper CODECs (COding-DECoding schemes/COder-DECoders). However it soon became clear that the real limit on communication rate was not set by the channel capacity, but by the cost of implementation of the CODECs. So it has been a point of constant research to invent and develop new refinements that will considerably reduce the system cost without bringing in any degradation in the performance of the system. The present work may also be identified as one such effort and is primarily aimed at improving the system performance by a technique that has been termed Multisampling, so that the performance of relatively simpler and hence cheaper systems can be improved to be at par with, if not better than the corresponding costlier counterparts.

An important point to be mentioned here is that the present thesis deals implicitly with coherent communication systems. It will become clear through the discussion and analysis to be presented that the technique of multisampling is meaningful only when there is a tight synchronization between the transmitter and the receiver.

1.1 Introduction to Multisampling and its Necessity:

In the conventional communication systems, one can model the receiver as a cascade of a bandpass RF/IF stage with a lowpass signal processing stage through a stage of coherent frequency translation. In some specific communication systems[1], there may be uncertainty in the signal level at the front-end of the bandpass stage. In such cases, it is a common practice to deliberately limit the received signal through saturating amplifier(s) in the bandpass stage. The deterioration in SNR (Signal-to-Noise-Ratio) due to such limiting in the bandpass stage has been treated extensively in the literature ([1],[5],[6]), with particular reference to low SNR applications. On the other hand, for nonfading high SNR channels, the bandpass front-end is linear and therefore does not have any saturating amplifier stage. A generalized receiver front end is shown in Fig. 1.1. The first block in this figure represents the bandpass stage as indicated. The bandpass stage is saturating (the bandpass limiter) when the communication is over fading channels, and is linear when it is over nonfading channels.

Due primarily to sampling rate requirements one cannot go in for digital signal processing in the bandpass stage, despite the multitude of advantages of the former over analog signal processing. In the lowpass stage, it is popular to use digital signal processing. As will be discussed in the next chapter, digital signal processing results in a considerable loss in the output SNR when the input SNR is low, especially when simple binary quantization is used. As a compromise, therefore, one is forced to use multilevel

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[1] The common examples are troposcatter and ionospheric links, satellite and microwave repeaters, and other fading channels.

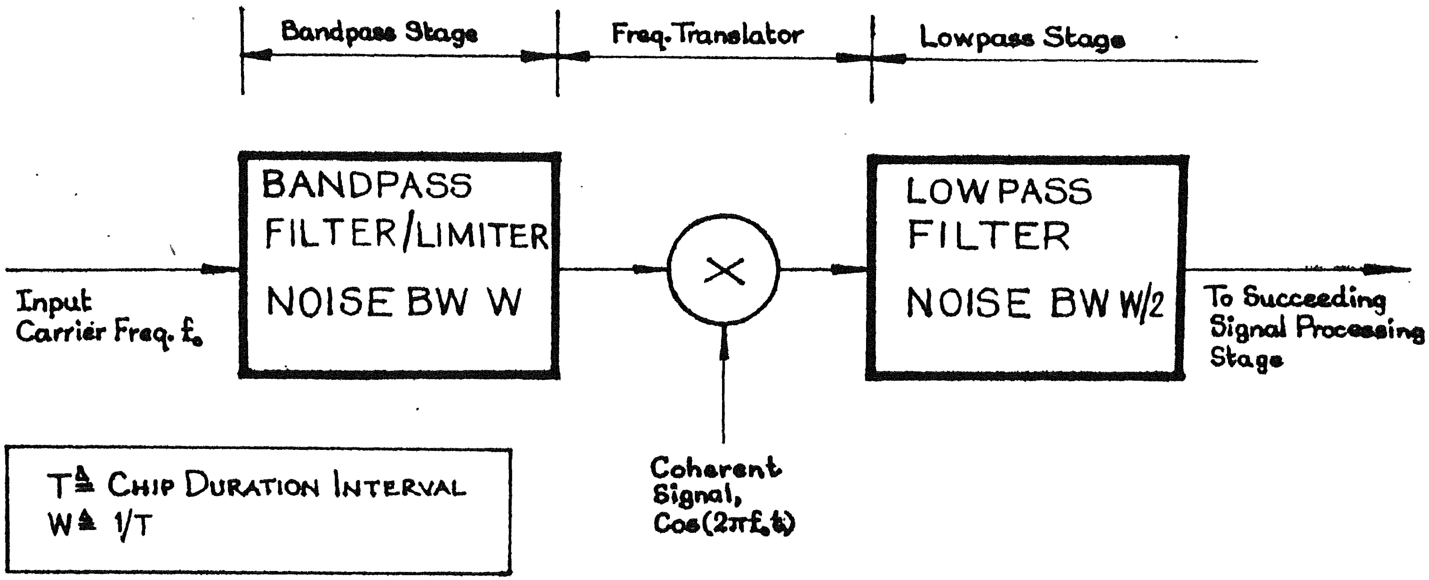

Figure 1.1 : Generalized Front End of a Data Communication Receiver.

quantization. But the price to be paid for this is the increase in the cost as well as the complexity. Multisampling, on the other hand, attempts to compensate for such SNR loss without going in for multilevel quantization, or any complex design of this kind, and hence promises a gross simplication as far as the lowpass stage is concerned.

But what is multisampling ? In simple terms it is a technique of gathering more information about a code chip than the existing methods of chip detection can provide under identical conditions. In the conventional systems, the output of the lowpass filter is sampled once per chip and the sample value represents the decision variable for the detection of the chip being processed. The sampling is to be done at what is called the optimum point. Multisampling, on the other hand, proposes sampling the chips at a multiple rate rather than taking only one sample per chip. As expected this should provide more information about the chip being sampled. This is indeed justified as proved analytically in the next chapters. Physically, the improvement comes as a result of the following process. Any probability density function that suits a realistic modelling of the noise process in the transmission channel will always have a nonzero probability of chip error, however large the SNR may be. However, in all such models, the probability of getting two samples in error is invariably less than that of one sample being in error. Probability of a larger number of samples being in error, is similarly further less. Thus it follows that with a suitable manipulation algorithm with the multiple samples one ought to achieve a less probability of error for the same operating SNR, or putting it in another way, the SNR output will be larger for a fixed input SNR had the technique of multisampling been resorted to in place of the conventional single sampling.

In purely qualitative terms, then, the major **advantages of** multisampling will be fully realized in cases where the 'effect' of the occurrence of an error in the single sample does not 'carry over' to the other samples to drive **them** into error. Technically speaking, this means that the autocovariance of the noise at the output of the lowpass filter,or more generally, the lowpass front end should be rather small in magnitude and also that the signal components of the samples should be nearly equal in value to each other. Now the transmitted chip pulse has a flat envelope. It is therefore required that the shape of the signal component of the output of the lowpass front end be as near to a rectangular pulse as possible. Due to the limited bandwidth, this criterion is seldom met. It is therefore a major design consideration to preserve the shape of the chip as accurately as possible over the domain of the sampling points inside the chip interval. From this argument the requirement of tight synchronization is readily appreciated. When the synchronization is somewhat loose, one must allow some tolerance in the positioning of the sampling points in a chip interval, and consequently the design of the filters now becomes even more difficult.

1.2 Previous Developments and Progress in the Analyses and Designs of Multisampled Digital Communication Systems:

Multisampling is relatively new concept first proposed by Das and Shanmugavel [3]. So far, they have been the only authors to report the principles and applications of multisampling ([3],[4]). They have mainly examined the improvement in performance that results when the decision variable for chip detection is formed by a linear sum of the multisamples. Theoretical

analyses have shown that [3] the SNR improvement with four samples per chip is in the range of 1 dB to 1·5 dB referred to single sampling. The different receiver configurations which they experimented with are summarily denoted by the generalized configuration shown in Fig. 1.2. The matched filters shown in this figure are for spread spectrum sequences and only spread spectrum communication has been examined in their work. The sampled data matched filter, which is shown in this figure will be formally discussed in chapter 2 in detail.

1.3 Foundations of the Present Work:

As discussed later, if there is no appreciable deviation of the chip shape from the ideal, the simple linear addition of the multisamples as done by Das and Shanmugavel is actually a discrete-time matched filtering operation on the chips. However a matched filter correlator is the optimum only when the additive noise accompanying the chips is white, i.e., the autocovariance of two distinct noise samples is identically equal to zero. In the actual situation, however, this requirement is seldom true owing to the limited bandwidth of the filters. Over a chip duration interval, the noise samples are far from being uncorrelated and the linear addition of the multiple samples is indeed suboptimum. The argument is further strengthened if the distortions in the chip shape are brought into consideration.

It is therefore imperative to form the decision variables by taking a weighted sum of the multiple samples rather than the simple linear sum. The choice of the weighting factors is governed by the chip shape after **the filtering operations** and the noise sample autocovariance. Both these factors depend on the filter characteristics. In

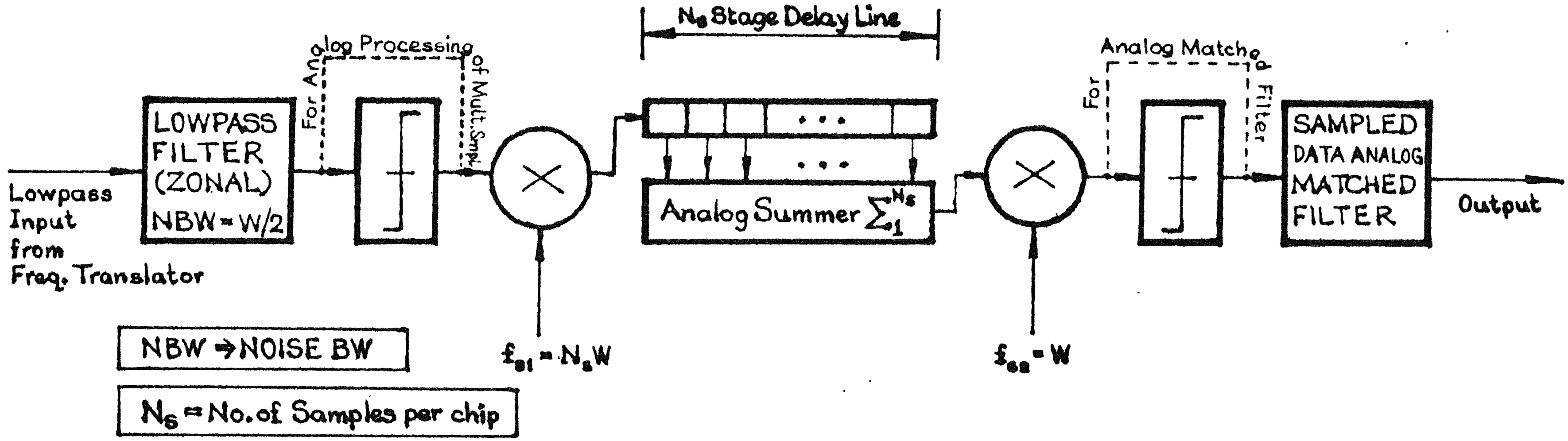

Figure 1.2 : Generalized Lowpass Stage of the Multisampled Receiver for Spread Spectrum Systems as in [3],[4]

chapter 3, the calculation of the optimum weighting factors in a generalized situation involving interchip interference as well as correlation of the additive noise is elaborated. Optimally weighted multisampling is henceforth termed as Adaptive Multisampling and the thesis focuses on the examination of and application of adaptive multisampling in various systems through theoretical as well as simulation studies.

1.4 Summary and Organization:

The thesis is organized in six chapters, including the present one. In chapter 2, an introduction to sampled data and digital matched filtering is provided. The evaluation of the minimum SNR necessary for reliable communication for various code rates using both hard and soft decision decoding is carried out to show theoretically that for very low SNRs, there is 2 dB loss in output SNR when hard decision is used, compared with soft decision. The rest of the chapter briefly reviews some of the calculations of [3] of importance to the discussions in the following chapters, and elaborates on the condition of zonality of the receiver filtering stages, - each in a separate section.

Chapter 3 begins with a detailed analysis of adaptive multisampling. Following this are presented some applications of adaptive multisampling in low rate coded communication systems. Altogether five receiver congifurations have been examined in this section, assuming zonal filters. The rest of the chapter studies the effects of nonzonal filtering and bandwidth variation on the performance of adaptive multisampling.

Chapter 4 reports the performance of adaptive multisampling in high rate coded communication systems. Several decoder configurations for the decoding

of convolutional codes have been studied assuming a zonal filter. A detailed discussion on nonzonal filtering with multisampling for such high-SNR applications covers the rest of the chapter.

Chapter 5 discusses the simulation methods adopted for the validation of the theoretical results. It also points out the weakness of such simulation algorithms in other important situations not studied in this thesis, and makes suggestions about other methods for the removal of such limitations.

The last chapter is the conclusion. After giving a brief overview of the important points discussed and the important results obtained in the thesis, it makes suggestions for the possibility of further work in this line.

CHAPTER 2: THE DIGITAL MATCHED FILTER AND RELATED TOPICS

ALTHOUGH the concept of matched filters is now over four decades old, their use till the last decade was confined mainly to large and expensive systems. One of the major reasons for this was the very characteristic which makes matched filters valuable - a large time-bandwidth (TW) product. With the advent of Surface Acoustic Wave Delay Lines (SAWDLs) and Large Scale Integration (LSI) technology, it is now commonplace to fabricate with high replicability analog and digital matched filters with TW products of several hundreds. As mentioned at the very beginning, a digital system has often several advantages over its analog counterpart, although a digitized receiver may show a poorer SNR performance compared to the analog one. The present chapter gives an introduction to the sampled data and digital matched filters and assesses the SNR loss due to pure hard decision. A brief review of the analyses put forward by Das and Shanmugavel for linear multisampling is presented next followed by a discussion on the zonal filter and zonality.

2.1 The Digital Matched Filter: [12]:

In analog matched filtering , the decision variable is computed by the correlation integral,

$$U = \int_0^{T_s} r(t) \cdot s(t) \cdot dt , \qquad (2.1.1)$$

where, $r(t)$ is the received waveform corresponding to the transmitted waveform $s(t)$, $0 \leqslant t < T_s$, T_s denoting the time interval for signalling. Eq. (2.1.1) can be written in the sampled data form as,

$$U = \delta \cdot \sum_{m=1}^{N} r(mT_s/N) \cdot s(mT_s/N)$$
$$= \delta \cdot \sum_{m=1}^{N} r^D(m) \cdot s^D(m) , \qquad (2.1.2)$$

where, δ is the sampling period and the superscript 'D' denotes discrete time operation. The sampling frequency $1/\delta$ must be at least at the Nyquist rate.

In digital matched filtering, the samples are normalized and quantized to a certain number of bits L. A normalized and quantized sample (or any normalized numerical quantity for that matter) may be expressed as,

$$x = \sum_{i=1}^{L} 2^{-(i-1)} \cdot x_{i-1} ,$$

where, x_i denotes the i^{th} significant bit. With this, the decision variable now becomes,

$$U = \sum_{i=1}^{L} \sum_{k=1}^{L} \sum_{m=1}^{N} 2^{-(i+k-2)} \cdot r_i^B(m) \cdot s_k^B(m) , \qquad (2.1.3)$$

where, $r_i^B(m)$ and $s_k^B(m)$ denote the i^{th} and k^{th} significant bits in the binary expansions of the quantized samples $r^D(m)$ and $s^D(m)$ respectively. Eq. (2.1.3) is the basic equation in the computation of the decision variables in digital matched filtering. For spread spectrum systems, N represents the length of the pseudonoise sequence and T_s denotes the duration of the N chips constituting the pseudonoise sequence.

It has been shown by Turin [12] that the computation using Eq. (2.1.3) will require L on-line shift-registers each of N stages, L replica code shift-registers also of N stages each, L^2N multipliers, and $L^2(N-1)$ adders

for the digital correlation circuit alone. Although LSI technology now is in a position to allow such a complex system to be integrated on one or a few LSI chips for moderate values of L and N, research has always been and is still under way to reduce system complexity without seriously degrading the performance of the system. Popularly, L is restricted to the value one, which is equivalent to performing hard decision decoding. In the next section, the disadvantages of hard decision in contrast with soft decision is discussed in terms of the SNR performance of the two.

2.2 Performance comparison of Systems with Hard Decision and Soft Decision:

Whenever binary quantization or hard decision is introduced in a digital communication system, there is a degradation in the output SNR. In the present section, a performance comparison between hard and soft decision is made assuming that the additive noise is Gaussian with zero mean.

The portion of the receiver of interest is shown in Fig. 2.1. For hard decision, the hard limiter is included in the circuit. Since the Gaussian density is a symmetrical probability density function about its mean the equivalent channel upto and including the hard limiter is a Binary Symmetric Channel (BSC). The BSC transition probability is given by,

$$p = \operatorname{erfc}(A_c/\sigma) \quad , \tag{2.2.1}$$

where A_c stands for the magnitude of the chip signal component at the instant of sampling, σ^2 is the variance of the additive Gaussian noise, $n'(t)$, assumed stationary at least

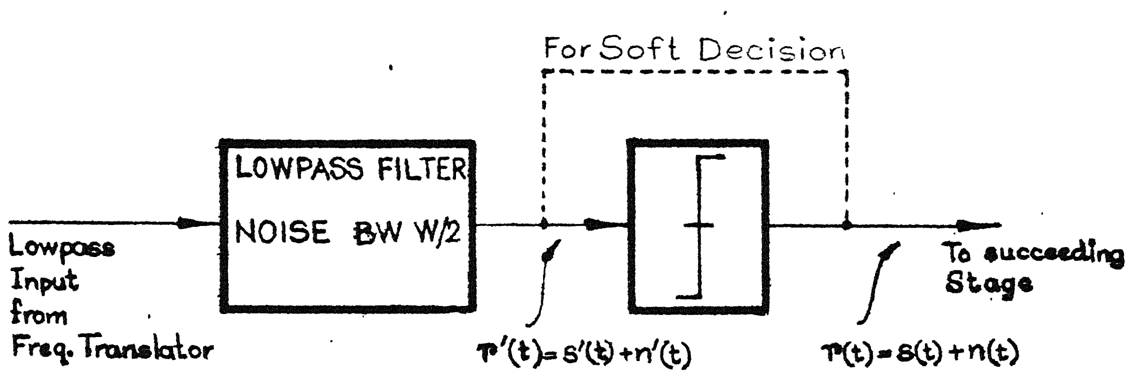

Figure 2.1 : Lowpass Front End Section for Hard Decision and Soft Decision.

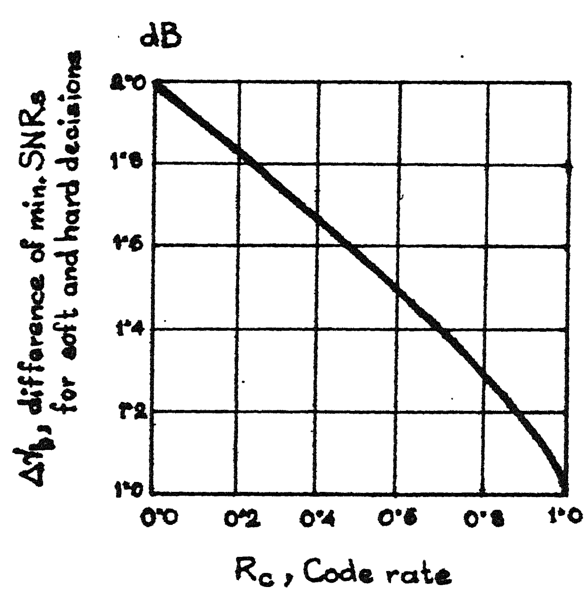

Figure 2.2 : Degradation due to Hard Decision as a Function of Code Rate.

in the wide sense, and where,

$$\operatorname{erfc}(x) = \frac{1}{\sqrt{2\pi}} \cdot \int_{x}^{\infty} \exp[-z^2/2] \cdot dz \,. \qquad (2.2.2)$$

The SNR per chip is given by,

$$\gamma_c = A_c^2/\sigma^2 \,, \qquad (2.2.3a)$$

and that per information bit by,

$$\gamma_b = \gamma_c/R_c = A_c^2/(\sigma^2 R_c) \,, \qquad (2.2.3b)$$

where, R_c denotes the rate of the code. For spread spectrum systems with a sequence length N, R_c is equal to 1/N.

In the appendix A-1, it has been shown that for code rates approaching zero, transmission at the channel capacity is possible with an arbitrarily small probability of error provided that the SNR per information bit is above a minimum value given by,

$$\gamma_b^H = \frac{\pi}{2} \cdot \log_e 2 \doteq 0{\cdot}37 \text{ dB} \,, \text{ for hard decision (BSC)}$$

$$\text{and, } \gamma_b^S = \log_e 2 \doteq -1{\cdot}60 \text{ dB} \,, \quad \text{for soft decision.} \qquad (2.2.4)$$

It is thus seen that for low rate codes, the degradation in the minimum SNR necessary with hard decisions is about 2 dB referred to soft decisions. Following the same basic method as in the appendix and using a numerical solution technique the values of γ_b^H and γ_b^S can be computed for other code rates and the result is shown in Fig. 2.2, which demonstrates the decibel difference between these two quantities. It is seen that as R_c approaches unity, the minimum necessary SNR in hard decisions is about 1 dB more than that in soft decisions.

The case of R_c approaching zero applies to the situation where N is very large, for example, as in a large length spread spectrum system, while that of R_c tending to unity applies in general to any high rate coded communication system, for example rate 2/3 or rate 3/4 convolutional codes, or even rate 1/2 or rate 1/3 convolutional codes. The 2 dB degradation at low code rates is a very well known result [6]. The fact that R_c is assumed equal to the channel capacity and simultaneously R_c is assumed to be approaching zero points to the particular situation where chip signalling is done with an extremely low SNR. For hard decisions this implies that the BSC transition probability is nearly 0·5. Under such circumstances, the asymptotic expression for the function erfc(x) can be used,

$$\mathrm{erfc}(x) \doteq \frac{1}{2} \cdot [1 - \sqrt{2/\pi}\, x] \qquad x \to 0 \quad . (2.2.5)$$

It is shown in appendix A-2 that for hard decision decoding, the spread spectrum matched filter correlator has an output SNR given by [4],

$$\gamma_b^H = N \cdot \frac{(1-2p)^2}{4p\cdot(1-p)} ,$$

which for p approaching 0·5, simplifies to,

$$\gamma_b^H \doteq N \cdot (1-2p)^2 . \qquad (2.2.6a)$$

The SNR output with soft decisions is given by[1],

$$\gamma_b^S \doteq N/\sigma^2 ; \qquad (2.2.6b)$$

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[1] Eq. (2.2.6b) is valid only when the noise components of the chips are uncorrelated. For bandlimited filtering, this is not met. However the equation is still valid if N is large.

Using Eq. (2.2.5) into (2.2.6), the loss due to hard decision is readily seen to be,

$$\Delta \gamma_b \doteq \pi/2 \doteq 2 \text{ dB},$$

which is the same as the previous result.

2.3 Nonadaptive (Suboptimum) Multisampling:

In this section, some of the theoretical results obtained by Das and Shanmugavel [3] will be reviewed very briefly for comparing with the results of adaptive multisampling in the following chapters. The theoretical gains as will be seen are rather inadequate to compensate for the SNR loss due to hard decision in spread spectrum systems. Two cases are considered, first with a lowpass limiter, and second with a simple lowpass filter at the front end of the lowpass stage. In both cases a perfect zonality is assumed.

Case I: With lowpass limiter.

In the case of a lowpass limiter, the output of the limiter will be nearly a random telegraph signal when the input SNR is low. As shown by Rice [10], the autocovariance between any two samples separated by a time τ is given by,

$$\varphi_r(\tau)=A_c^2 \cdot \exp[-1{\cdot}155W|\tau|] \tag{2.3.1}$$

where A_c is the level of the lowpass limiter output and W is the inverse of the chip duration interval, T. Thus W is equal to double the maximum fundamental frequency possible among different chip trains. The lowpass filter is assumed to have a bandwidth of W/2 - just sufficient to pass this maximum fundamental frequency, and is assumed to have an

ideal rectangular frequency response characteristic with linear phase. The variance of the sum of the multisamples in the case of quadruple (fourfold) sampling is easily shown from Eq. (2.3.1) to be,

$$\sigma_4^2 = A_c^2 \cdot [4 + 2(0 \cdot 75 + 0 \cdot 56 + 0 \cdot 42 + 0 \cdot 56 + 0 \cdot 75 + 0 \cdot 75)]$$

$$= 11 \cdot 58 A_c^2 ,$$

while by the assumption of zonality, the signal power of the sum is given by,

$$P_4^S = 16 A_c^2 .$$

Thus the multisampler output has an SNR that is $16/11 \cdot 58 = 1 \cdot 38$ times the hard limiter output for single sampling. Thus a gain of about $1 \cdot 4$ dB over single sampling is resulted,

$$\Delta \, SNR_{1x}^{NAMx} = 1 \cdot 4 \text{ dB} . \qquad (2.3.2)$$

Case II: Without lowpass limiter (linear front end).

In this case, the autocovariance of the noise component is given by (assuming the same type of low-pass front end filter),

$$\varphi_r(\tau) = \varphi_r(0) \cdot \frac{\mathrm{Sin}[\pi W \tau]}{\pi W \tau} , \qquad (2.3.3)$$

and a similar procedure assuming perfect zonality will give for quadruple sampling,

$$\Delta \, SNR_{1x}^{NAMx} = 1 \cdot 05 \text{ dB} . \qquad (2.3.4)$$

Neither of these results is near the desired figure of 2 dB. So ordinary (nonadaptive) multisampling cannot fully compensate for the SNR loss due to hard decisions.

2.4 The Zonal Filter:

The terms zonal filter and zonality have been repeatedly spoken of in the previous discussions. As defined earlier a zonal filter is a bandlimiting (bandpass or lowpass) filter which while keeping a desired noise bandwidth, imparts little or no changes in the chip shapes, at least over the span of the sampling point(s) inside the chip interval. In the present section, the characteristics of such a filter is briefly discussed, without going into the details about any particular filter transfer function(s).

For communication over fading channels, it is a common practice to limit the amplitude of the received signal in the bandpass stage by using what is called a bandpass limiter, as discussed in section 1.1.Assuming a low SNR at the input and rectangular power spectra for both the signal and the additive noise at the input to the bandpass limiter, Cahn [1] has evaluated the SNR loss for various ratios of the bandwidth of the signal to that of the noise. The input to the bandpass limiter is usually obtained from the RF front end bandlimiting stage and one can take this ratio to be approximately unity. The SNR loss for this case is the minimum and equals about 0·5 dB. Now, the bandlimiting gives rise to intersymbol interference, or more correctly, what may be called interchip interference. Further interference may be introduced by the lowpass filter following the frequency translator. If $A(t)$ denotes the time variant envelope of the IF pulses at the output of the bandpass limiter (PSK signalling is assumed) for $0 \leq t < T$, where T is the chip sig-

nalling interval, then the gain in the SNR due to a coherent frequency translation is given by,

$$\Delta\, \mathrm{SNR}_{BP}^{LP} = \frac{\int_0^T A^2(t) \cdot dt}{\int_0^T A^2(t) \cdot \mathrm{Cos}^2[2\pi f_o t] \cdot dt} , \tag{2.4.1}$$

in which f_o denotes the centre frequency of the bandpass (IF) stage. For perfect zonality, $A(t)$ would be a constant over the chip signalling interval and the SNR gain due to coherent frequency translation would be at its maximum value of 3 dB. And under such cases, the overall SNR gain would be 2·5 dB, by taking into account the SNR loss due to the bandpass limiter. For low SNR applications using large length spread spectrum modulation, the input chip SNR is usually less than -10dB and even after the 2·5 dB improvement, the SNR remains low. As discussed by Cahn [1] and also by Davenport [6], the noise at the output of the lowpass filter is approximately Gaussian. These informations will be necessary while discussing zonal multisampling both in this section and elsewhere.

To start the discussion on zonality, first analog multisampling is considered. By analog multisampling one means an analog processing of the multisamples. As pointed out, the accompanying noise at the lowpass filter output is Gaussian. For the criterion of zonality to be satisfied, the bandpass and lowpass filters must both possess a nonzero frequency response for at least a minimum frequency of $N_s W/2$ relative to the respective centre frequencies, where N_s is the number of samples taken per chip. Now in all practical communication systems, the filtered chips at the output of the receiver lowpass filter can at the worst look like an approximate trapezoid with the corners smoothed out. For quadruple sampling the multisamples are almost zonal in such cases and thus zonality is not an impractical proposition. It is not

however the aim of the present thesis to investigate the exact transfer functions of the filters. It is tacitly assumed therefore that all the filters involved are zonal, unless specified otherwise. However, in discussing analytically the performance of analog multisampling one has to choose a definite transfer function to start with. It is therefore selected as a rule that the zonal filters have a Gaussian frequency response characteristic, while the nonzonal ones have a rectangular frequency response characteristic - both with linear phase. It is admitted that the Gaussian filters are themselves nonzonal and therefore assuming zonality with the Gaussian filter is only of academic interest. However, it is emphasized that the choice is governed by an easy mathematical tractability of this transfer function so that the merit of multisampling can be appreciated both theoretically and practically (i.e., through simulation).

Next, digital multisampling is taken up. By digital multisampling is meant a digital (quantized) processing of the multisamples. This is done by using a lowpass limiter in place of the lowpass filter, and taking samples from this limiter. As discussed in section 2.2, this will result in a 2 dB loss in the output SNR referred to the linear processor when the input SNR is low. As the SNR in the lowpass front end is low and the accompanying noise is Gaussian, it follows that the lowpass limiter output will be like a random telegraph signal almost independent of the signal values in the multisamples. Thus small variations in the signal magnitudes will be suppressed by the limiter and in effect, the filters will be as good as zonal filters[2].

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[2]This is due to the slow variation of erfc(x) with x at smaller values of x. If the SNR is high and/or there is large interchip interference, this will no longer be true.

It is then logical to expect that digital multisampling will be less sensitive to interchip interference than its analog counterpart, provided of course that the input SNR is low and also that the amount of interchip interference is not very large. This is why digital multisampling has been treated in equal detail as analog multisampling in connection with large length spread spectrum systems in the next chapter.

2.5 Summary:

In this chapter the preliminaries have been discussed in the necessary details. First sampled data and digital matched filters have been discussed. This was followed by a comparison of hard and soft decoding in terms of their SNR performance. Following this has been a brief review of some important results of the work done in this field. Lastly the zonal filter and zonality have been discussed. It has been reviewed in this chapter that hard decoding is indeed worse compared to soft decoding, and for low SNR applications, it has been analytically revalidated that the available output SNR in hard decision decoding is always 2 dB less than that in soft decision decoding.

CHAPTER 3: INTRODUCTION TO ADAPTIVE MULTISAMPLING AND ITS APPLICATIONS IN SPREAD SPECTRUM DIGITAL COMMUNICATION SYSTEMS

IN ordinary nonadaptive multisampling that has been proposed by Das and Shanmugavel ([3],[4]), the basic assumption is that of a zonal filter. However their scheme is suboptimum, as pointed out in the previous chapters, when the filters induce interchip interference and when the auto-covariance of the noise at the multisampler input has nonzero values. Under such circumstances, a weighted sum of the multi-samples would be the optimum way of forming the decision variables provided of course that the weighting factors are chosen optimally. In the present chapter the theory of this kind of adaptive multisampling scheme has been studied and its merits discussed with regard to low rate spread spectrum communication systems.

3.1 Principles of Adaptive Multisampling:

A multisampler can be looked upon as a sampled data filter that is used to process each individual chip independently of the others. Ordinary nonadaptive multisampling is nothing but the matched filter for the flat pulse. Even when the filters are zonal, this will not be the optimum unless the noise accompanying the signal at the input to the multisampler is uncorrelated. For bandlimited filters neither zonality nor uncorrelatedness of the noise is truly satisfied, and for the common types of filters demonstrating approximate zonality over the span of the multisampling points the noise is seldom uncorrelated. So the nonadaptive multi-sampling is indeed suboptimum as will be proved analytically as well as experimentally (simulation) in this and the sub-sequent sections. In general, optimum multisampling refers

to that kind of processing of the multiple samples which will maximize the output chip SNR for a given input chip SNR. Its algorithm can be designed only when the noise autocovariance as well as the exact signal shape at the input to the multisampler are known completely. Kumar [8] has discussed these aspects in detail in connection with optical cross correlators. The approach adopted here bears close similarity with his.

Let $h^A(t)$, $0 \leq t < T$, denote the continuous-time impulse response of a linear filter. The filter has a continuous-time input signal $r(t)$, $0 \leq t < T$, giving the output $z^A(t)$, $0 \leq t < T$, which is also continuous in time over the interval of interest. The corresponding discrete-time sequences are denoted by $h(k)$, $y(k)$ and $z(k)$, $k=1,2,\cdots,N_s$, where, N_s is as before the number of multiple samples taken per chip. Thus,

$$\begin{aligned} h(k) &= h^A(kT/N_s)\ , \ k = 1, 2, \cdots, N_s\ , \\ y(k) &= r(kT/N_s)\ , \ k = 1, 2, \cdots, N_s\ , \\ z(k) &= z^A(kT/N_s)\ , \ k = 1, 2, \cdots, N_s\ . \end{aligned} \tag{3.1.1}$$

In this manner the multisampler has been identified with a discrete-time version of a linear filter. Henceforth adaptive multisampling and nonadaptive (ordinary) multisampling will be denoted respectively as AMx and NAMx for the sake of brevity. The output SNR of the multisampler is defined as,

$$\mathrm{SNR} = \frac{|E[z(N_s)]|^2}{\mathrm{Var}[z(N_s)]}\ . \tag{3.1.2}$$

The sequence $z(i)$ is given by the convolution of the sequences $y(i)$ and $h(i)$, i.e.,

$$z(i) = y(i) \circledast h(i) = \sum_{k=1}^{N_s} y(k) \cdot h(i-k+1), \tag{3.1.3}$$

where, the symbol $\odot$ is used to denote the discrete convolution operation. Denoting the signal and noise components of y(k) by s(k) and n(k) respectively, and putting $i=N_s$ in Eq. (3.1.3) it follows that,

$$z(N_s) = \sum_{k=1}^{N_s} s(k) \cdot h(N_s-k+1) + \sum_{k=1}^{N_s} n(k) \cdot h(N_s-k+1) \, ,$$

which upon taking expected values yields,

$$E[z(N_s)] = \sum_{k=1}^{N_s} s(k) \cdot h(N_s-k+1) \, . \qquad (3.1.4)$$

Further the variance of $z(N_s)$ is given by,

$$\begin{aligned} \mathrm{Var}[z(N_s)] &= E[(z(N_s))^2] - [E(z(N_s))]^2 \\ &= \sum_{i=1}^{N_s} \sum_{k=1}^{N_s} E[n(i)n(k)] \cdot h(N_s-i+1) \cdot h(N_s-k+1) \\ &= \sum_{i=1}^{N_s} \sum_{k=1}^{N_s} h(N_s-i+1) \cdot \varphi(i,k) \cdot h(N_s-k+1) \, , \end{aligned} \qquad (3.1.5)$$

where, $\varphi(i,k)$ stands for the autocovariance function of the noise samples n(i) and n(k). Assuming the noise to be stationary at least in the wide sense,

$$\varphi(i,k) = \varphi(k,i) = \varphi(|i-k|) = \varphi_r\left[\frac{i-k}{N_s} \cdot T\right] \, , \qquad (3.1.6)$$

where, $\varphi_r(\tau)$ is the autocovariance of the noise in the continuous-time received signal r(t). Defining,

$$g(k) = h(N_s-k+1) \ , \quad k = 1, 2, \cdots, N_s \ , \qquad (3.1.7)$$

the SNR equation becomes,

$$\mathrm{SNR} = \frac{\left|\sum_{k=1}^{N_s} g(k)\cdot s(k)\right|^2}{\sum_{i=1}^{N_s}\sum_{k=1}^{N_s} g(i)\cdot\varphi(i,k)\cdot g(k)} \qquad (3.1.8)$$

Using the matrix notations,

$$G = [g(1) \quad g(2) \quad g(3) \quad \cdots \quad g(N_s)]^T \ , \qquad (3.1.9a)$$

$$S = [s(1) \quad s(2) \quad s(3) \quad \cdots \quad s(N_s)]^T \ , \qquad (3.1.9b)$$

$$R = \begin{bmatrix} \varphi(1,1) & \varphi(1,2) & \varphi(1,3) & \cdots & \varphi(1,N_s) \\ \varphi(2,1) & \varphi(2,2) & \varphi(2,3) & \cdots & \varphi(2,N_s) \\ \varphi(3,1) & \varphi(3,2) & \varphi(3,3) & \cdots & \varphi(3,N_s) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \varphi(N_s,1) & \varphi(N_s,2) & \varphi(N_s,3) & \cdots & \varphi(N_s,N_s) \end{bmatrix} , \qquad (3.1.9c)$$

Eq. (3.1.8) can be compactly written as,

$$\mathrm{SNR} = |G^T S|^2/[G^T R G] \qquad (3.1.10)$$

Differentiating Eq. (3.1.8)with respect to the different $g(k)$, and equating each to zero, one can show that the condition for the maximum SNR can be written as, (See also p. 49)

$$G = \alpha\varphi(0)\cdot R^{-1}S \triangleq G_o \ , \qquad (3.1.11)$$

where, $\alpha\varphi(0)$ is a scalar constant multiplier used for the purpose of normalization wherever necessary, and $\varphi(0)$ is the variance of the individual noise samples, denoted by σ^2 in the earlier chapters. The elements of the matrix G_o denoted by $g_o(k)$, $k=1,2,\cdots,N_s$, represent the weighting

factors for the multisamples. Thus the matrix G_o,

$$G_o = [g_o(1) \quad g_o(2) \quad g_o(3) \quad \cdots \quad g_o(N_s)], \quad (3.1.12)$$

may be called the coefficient matrix. The coefficient $g_o(k)$ represents the optimum weight to be associated with the k^{th} multisample prior to addition.

For single sampling, the sampling is done at the optimum point inside the chip interval, i.e., at the point where the signal component of the chip reaches its maximum value. For the usual type of interchip inteference the optimum single sampling point falls either at the end of the chip interval (for causal front end filter) or at the centre of the chip interval (for noncausal front end filter). In either case, the single sample is assumed to have a signal component henceforth denoted by $s(k_o)$. It is to be noted that in the last notation, k_o may not always be an integer if the definition, $s(m)=s^A[mT/N_s]$, is stuck to. Needless to mention that $s^A(t)$ denotes the signal component of the continuous-time waveform $r(t)$, $0 \leq t < T$. The SNR output of the single sampler is then defined as,

$$SNR_{1x} = |s(k_o)|^2/\varphi(0) \quad . \quad (3.1.13)$$

With adaptive multisampling using weights given by Eq. (3.1.12) the SNR output is easily obtained as,

$$SNR_{AMx} = S^T R^{-1} S \quad . \quad (3.1.14)$$

Thus the advantage in SNR due to AMx over single sampling is,

$$\Delta SNR_{1x}^{AMx} = \frac{SNR_{AMx}}{SNR_{1x}} = \varphi(0) \cdot [S^T R^{-1} S]/|s(k_o)|^2 \quad (3.1.15)$$

For NAMx, the coefficient matrix is proportional to S, and so,

$$G = \alpha \cdot S = G_{MF} \quad , \tag{3.1.16}$$

which results in an output SNR given by,

$$SNR_{NAMx} = |S^T S|^2/[S^T RS] \quad . \tag{3.1.17}$$

The SNR advantage due to ordinary multisampling over single sampling is therefore,

$$\begin{aligned} \Delta\, SNR_{1x}^{NAMx} &= \frac{SNR_{NAMx}}{SNR_{1x}} \\ &= \varphi(0) \cdot |S^T S|^2/([S^T RS] \cdot |s(k_o)|^2) \quad , \end{aligned} \tag{3.1.18}$$

and that due to adaptive multisampling over ordinary multisampling is,

$$\begin{aligned} \Delta\, SNR_{NAMx}^{AMx} &= \frac{SNR_{AMx}}{SNR_{NAMx}} \\ &= [S^T RS] \cdot [S^T R^{-1} S]/|S^T S|^2 \quad . \end{aligned} \tag{3.1.19}$$

It is to be noted that $\Delta\, SNR_{NAMx}^{AMx} \geqslant 1$, and the equality holds if and only if R is an identity matrix possibly with a scalar multiplier, i.e., if and only if, the noise samples are fully uncorrelated. Thus with correlated noise adaptive multisampling is always advantageous over ordinary or nonadaptive multisampling.

With this introduction, some applications of adaptive multisampling will now be discussed in the coming sections. The systems considered are mostly large length spread spectrum receiver using pseudonoise sequences.

3.2 Applications: Adaptive Multisampling in Spread Spectrum Digital Communication Receivers:

In spread spectrum systems, it is a usual practice to use a bandpass limiter in the bandpass stage for reasons outlined in the earlier chapters, particularly in section 1.1. For large length systems, the input SNR is low and therefore the output noise from the bandpass limiter has a Gaussian density with zero mean, as discussed earlier in section 2.4. For the lowpass filter output noise, the Gaussian nature is maintained [15] but the power spectrum gets further modified by the frequency response characteristics of the lowpass filter. In the present section both analog and digital multisampling are examined with regard to large length spread spectrum systems. For the former, the lowpass front end filters shown in the diagrams are assumed to represent the lowpass equivalent for the entire bandpass and lowpass filtering stages, while for the latter the lowpass front end filters are assumed to denote what they are in the lowpass stage only. Due to such assumptions, the input noise at the lowpass front end can be taken as white for analog multisampling, while that for digital multisampling is assumed derived from the bandpass limiter stage as usual.

3.2.1 Adaptive Analog Multisampling with Zonal Filters:

In this subsection only zonal filter is considered and as argued in section 2.4, the filter is assumed to have a Gaussian frequency response, with linear phase. Two different receiver structures are examined; the first is a fully linear (analog) configuration, while the second introduces digital matched filtering in place of the analog one.

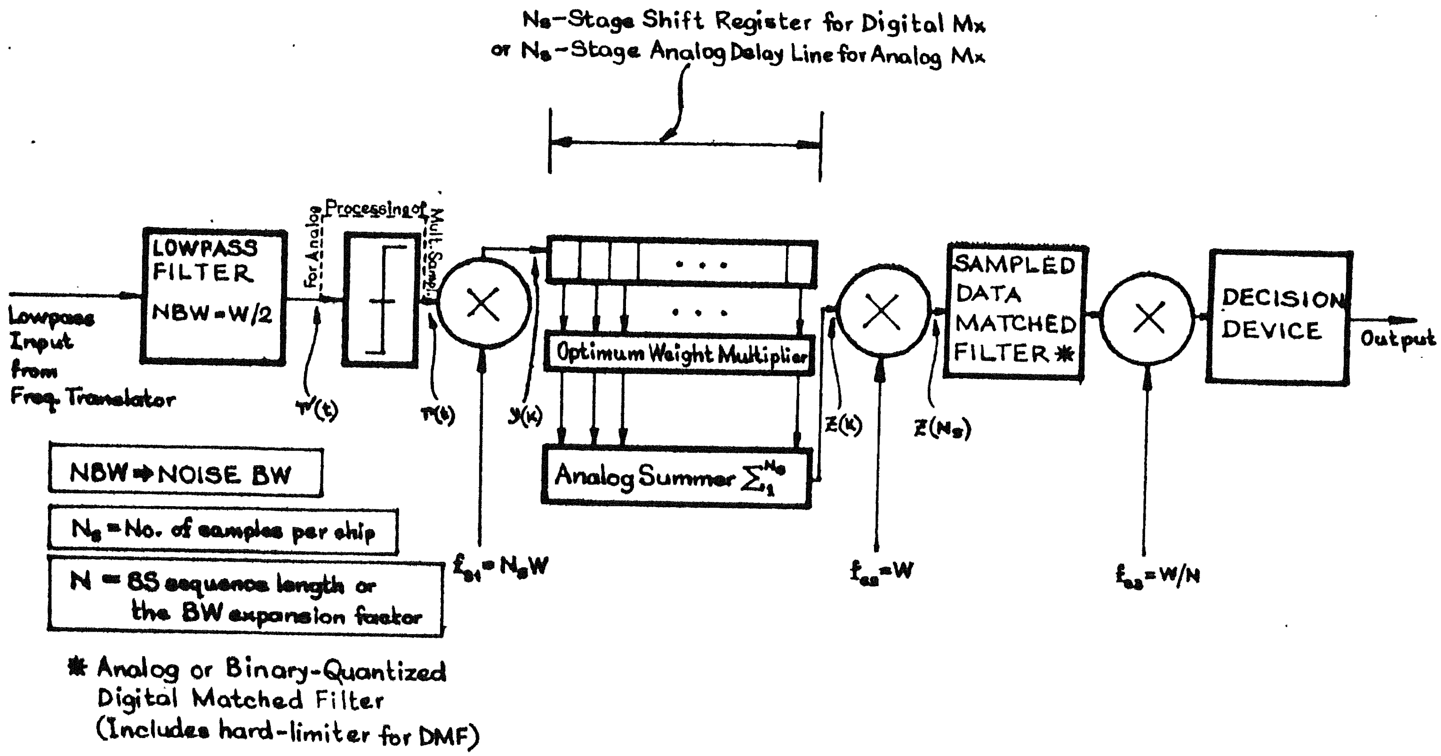

Figure 3.1 : Generalized Lowpass Stage of the Multisampled Receiver for Spread Spectrum Systems showing Adaptive Weighting in a Distinct Block.

(A) The Linear Circuit.

The representative block diagram is shown in Fig. 3.1 with the front end hard limiter replaced by a short and an analog matched filter used in place of the correlation circuit. The delay lines in the multisampler are nowadays conveniently realized using Charge-Coupled-Devices (CCDs) or Bucket-Brigade-Devices (BBDs). For the Gaussian type of transfer function, the output noise from the lowpass filter has an autocovariance function given by [13],

$$\varphi_r(\tau) = \frac{\sqrt{\pi} \cdot f_c}{\sqrt{\log_e 2}} \cdot \exp[-(\pi f_c|\tau|)^2/(4\log_e 2)] \quad ,(3.2.1)$$

where f_c denotes the 3 dB cutoff frequency of the filter and is taken to be equal to W/2, i. e., the desired noise-bandwidth[1]. Then the noise autocovariance matrix R with a quadruple sampling becomes,

$$R = \varphi(0) \begin{bmatrix} 1\cdot000 & 0\cdot801 & 0\cdot411 & 0\cdot135 \\ 0\cdot801 & 1\cdot000 & 0\cdot801 & 0\cdot411 \\ 0\cdot411 & 0\cdot801 & 1\cdot000 & 0\cdot801 \\ 0\cdot135 & 0\cdot411 & 0\cdot801 & 1\cdot000 \end{bmatrix} , \quad (3.2.2)$$

whose inverse is,

$$R^{-1} = \frac{1}{\varphi(0)} \begin{bmatrix} 6\cdot593 & -10\cdot900 & 8\cdot790 & -3\cdot516 \\ -10\cdot900 & 22\cdot855 & -20\cdot833 & 8\cdot790 \\ 8\cdot790 & -20\cdot833 & 22\cdot855 & -10\cdot900 \\ -3\cdot516 & 8\cdot790 & -10\cdot900 & 6\cdot593 \end{bmatrix} \quad (3.2.3)$$

[1]Actually with f_c=W/2, the noise bandwidth of the lowpass filter becomes 0·53W, which is slightly (about 6 percent) more than the desired value.

In Eqs. (3.2.2) and (3.2.3), $\varphi(0)$ denotes the noise variance σ^2 as before. Using now the assumption of zonality, all the signal components are set equal to A_c, and thus the normalized optimum weighting coeffinients are obtained as, [Eq. (3.1.12)]

$$\begin{aligned} g_o(1) &= g_o(4) = 1\cdot000 \,, \\ g_o(2) &= g_o(3) = -0\cdot091 \,. \end{aligned} \tag{3.2.4}$$

The advantage with AMx over single sampling is, [Eq. (3.1.15)]

$$\Delta\, \mathrm{SNR}_{1x}^{AMx} \doteq 2\cdot45 \text{ dB} \,, \tag{3.2.5}$$

while that over NAMx is, [Eq. (3.1.19)]

$$\Delta\, \mathrm{SNR}_{NAMx}^{AMx} \doteq 0\cdot71 \text{ dB} \,.$$

The gain that would be obtained over single sampling had NAMx been used in place of AMx is, [Eq. (3.1.17)]

$$\Delta\, \mathrm{SNR}_{1x}^{NAMx} \doteq 1\cdot74 \text{ dB} \,. \tag{3.2.6}$$

In the simulation experiment with AMx, however, the SNR gain over single sampling was approximately 2·1 dB instead of the theoretical figure of 2·45 dB given earlier. The experimental results are presented in Fig. 3.2.

(B) The Digital Matched Filter Receiver.

This configuration differs from the previous one in that here in this circuit, a hard quantized (L=1) digital matched filter is used in place of the analog matched filter. According to the discussions in section 2.2 the performance of this configuration will be 2 dB worse in

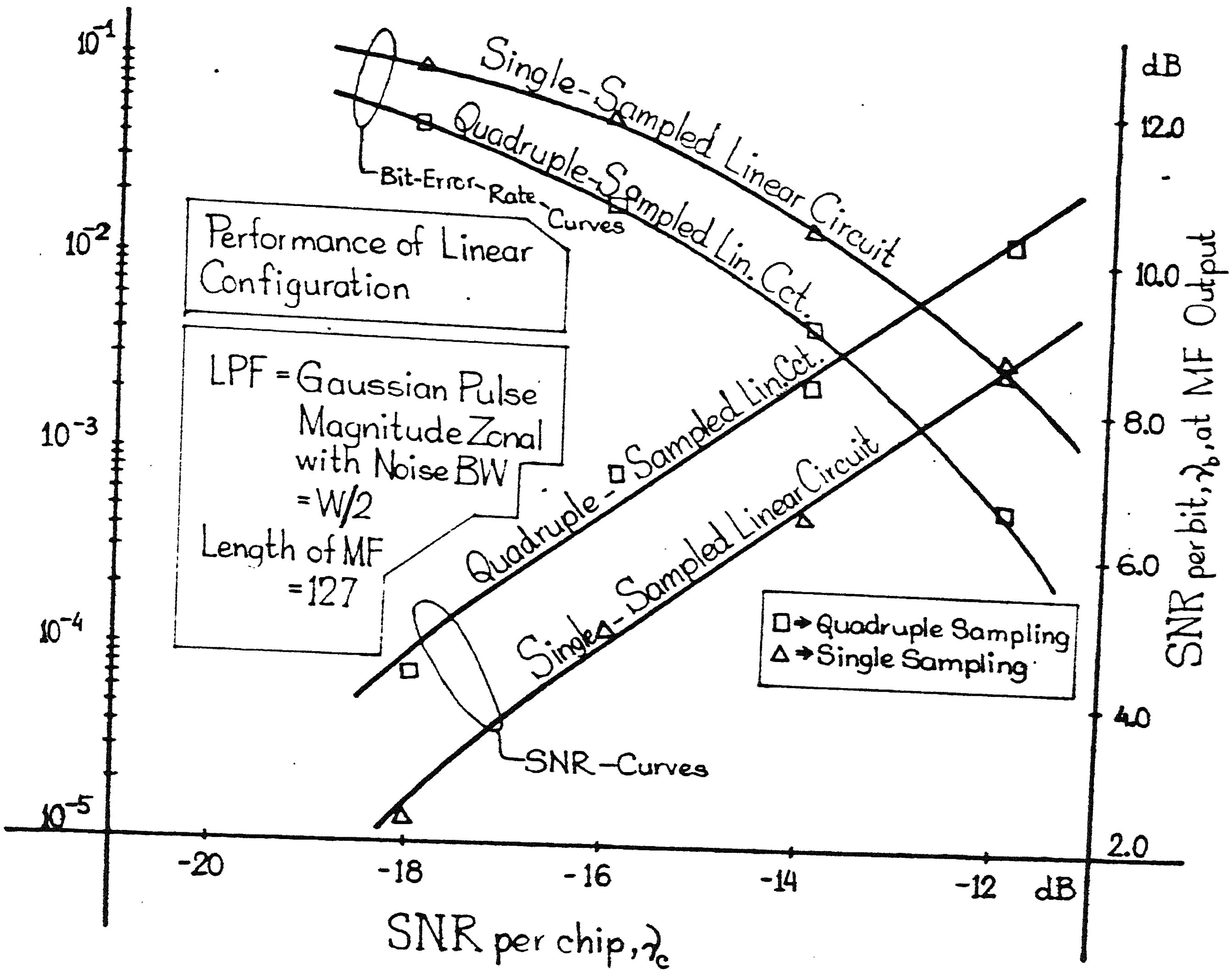

Figure 3.2 : Performance of Adaptive Analog Multisampling with Zonal Filters using the Fully Linear Configuration for Spread Spectrum Systems.

terms of SNR output compared with the previous one for single as well as multiple sampling. This is shown in Fig. 3.3, in the experimental curves. However the main point of interest is the fact that the quadruple sampled digital matched filter configuration can nearly coperform the singly sampled linear configuration. This is enough for concluding that multisampling with a zonal filter can adequately replace multilevel quantization and thus result in considerable simplification of the receiver organization without yielding to any degradation in the performance, compared with the conventional single sampled linear configuration.

3.2.2 Adaptive Digital Multisampling with Zonal Filters:

This subsection is devoted to the study of digital multisampling with the zonal filter assumption. The filter transfer function is assumed to have a Gaussian nature with linear phase as in analog multisampling . Now , as discussed in section 2.4, the dependence of SNR performance of digital multisampling on the interchip interference is less than that in analog multisampling, and therefore, the exact nature of the filter transfer function does not modulate the performance appreciably as long as the input SNR is low. In the following three receiver configurations have been studied; first, the analog matched filter with front end lowpass limiter, second, the digital matched filter with front end lowpass limiter, and finally, a special type of receiver organization consisting of independent parallel digital matched filters for the parallel processing of the multisamples of a chip. In the last configuration just the idea is to process the corresponding multisamples in the entire spread spectrum chip sequence through a dedicated digital matched filter of the parallel ones. Each set of corresponding multisamples (there can be only N_s number of such sets) is thus allotted

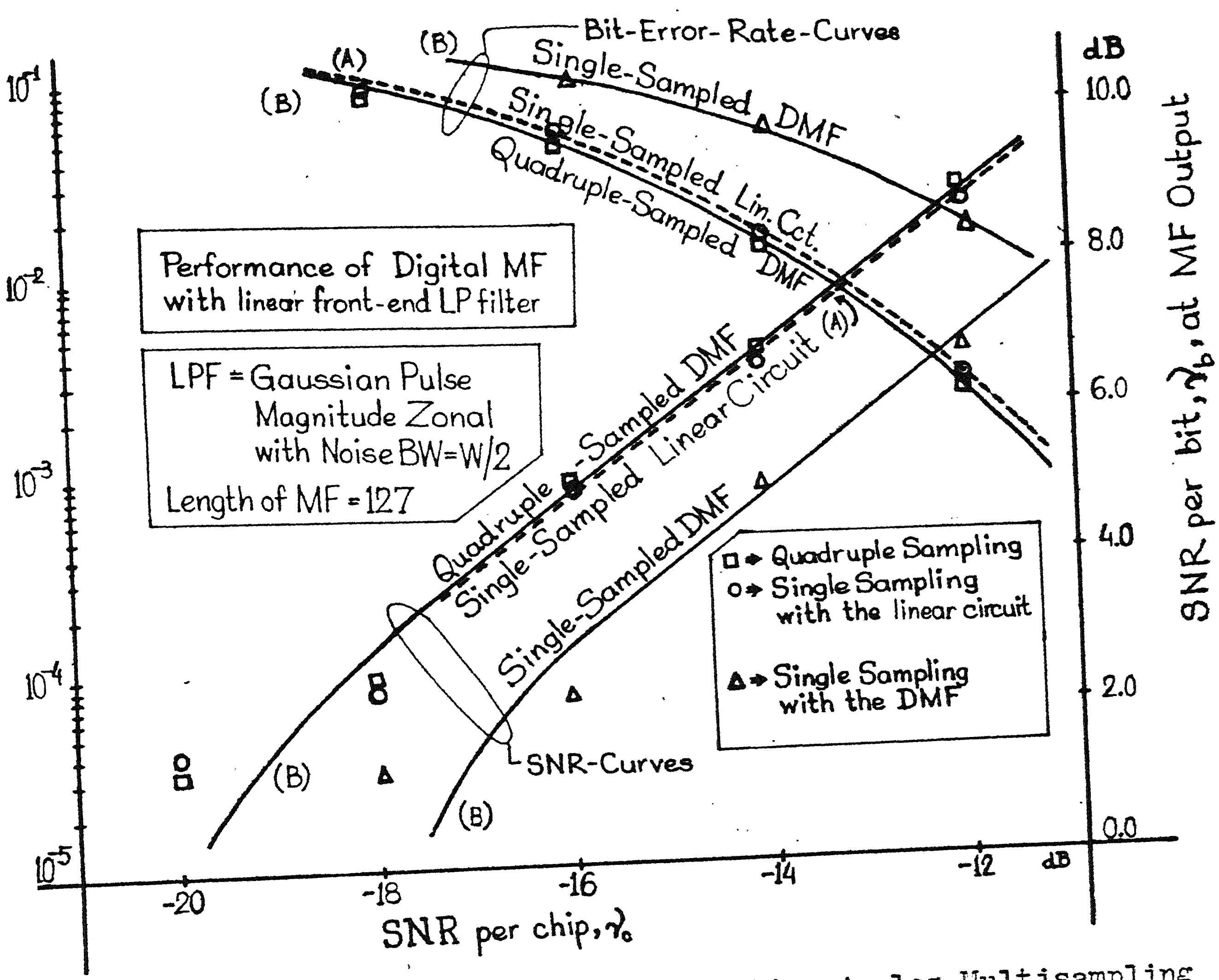

Figure 3.3 : Performance of Adaptive Analog Multisampling with Zonal Filters using the Digital Matched Filter for Spread Spectrum Systems.
(A) Single Sampled Linear Configuration,
(B) The Present Configuration.

a separate DMF 'of its own'.

(A) The Analog Matched Filter with Lowpass Limiter.

This is represented once again by Fig. 3.1 but now the hard limiter at the front end is included in the circuit and the matched filter is analog. As discussed in sections 2.2 to 2.4, the hard limiter output r(t) is just like a random telegraph signal, nearly independent of the interchip interference and the signal strength, provided that the SNR is low. The noise at the hard limiter output has an autocovariance function given by [10],

$$\varphi_r(\tau) = A_c^2 \cdot \exp[-1\cdot155W\cdot|\tau|] \quad . \tag{3.2.7}$$

Assuming a quadruple sampling, the noise autocovariance matrix becomes,

$$R = \varphi(0)\begin{bmatrix} 1\cdot000 & 0\cdot749 & 0\cdot561 & 0\cdot421 \\ 0\cdot749 & 1\cdot000 & 0\cdot749 & 0\cdot561 \\ 0\cdot561 & 0\cdot749 & 1\cdot000 & 0\cdot749 \\ 0\cdot421 & 0\cdot561 & 0\cdot749 & 1\cdot000 \end{bmatrix} , \tag{3.2.8}$$

whose inverse is,

$$R^{-1} = \frac{1}{\varphi(0)}\begin{bmatrix} 1\cdot979 & -1\cdot307 & 0\cdot002 & -0\cdot003 \\ -1\cdot307 & 2\cdot733 & -1\cdot316 & 0\cdot002 \\ 0\cdot002 & -1\cdot316 & 2\cdot733 & -1\cdot307 \\ -0\cdot003 & 0\cdot002 & -1\cdot307 & 1\cdot979 \end{bmatrix} , \tag{3.2.9}$$

where $\varphi(0)=A_c^2$. Now setting all the signal components s(k), k=1,2,3,4 to A_c , the normalized optimum weighting factors are obtained as, [Eq. (3.1.12)]

$$g_o(1) = g_o(4) = 1\cdot000 \;, \qquad\qquad (3.2.10)$$
$$g_o(2) = g_o(3) = 0\cdot167 \;,$$

and the advantage with AMx over single sampling, [Eq. (3.1.15)]

$$\Delta\, SNR_{1x}^{AMx} \doteq 1\cdot94 \text{ dB} \doteq 2 \text{ dB} \;, \qquad\qquad (3.2.11)$$

while that over NAMx is, [Eq. (3.1.19)]

$$\Delta\, SNR_{NAMx}^{AMx} \doteq 0\cdot54 \text{ dB} \;.$$

The gain with NAMx over single sampling is, [Eq. (3.1.17)]

$$\Delta\, SNR_{1x}^{NAMx} \doteq 1\cdot40 \text{ dB} \;,$$

which is the same as obtained by Das and Shanmugavel [3], and also as reviewed in section 2,3 from another approach.

It is seen from Eq. (3.2.11) that the AMx provides a gain that is just sufficient to compensate for the 2 dB loss due to the front end digitization in the lowpass stage. Thus it is concluded that a quadruple sampled analog matched filter with lowpass limiter is able to coperform the corresponding singly sampled linear configuration presented earlier. The experimental performance figures are shown in Fig. 3.4.

(B) The Single Digital Matched Filter with Lowpass Limiter.

In this case the receiver configuration can be obtained once again from Fig. 3.1, by including the **lowpass limiter** as in the last case and using a digital matched filter (hard quantized) in place of the correlation circuit. The incorporation of the extra hard quantizer for

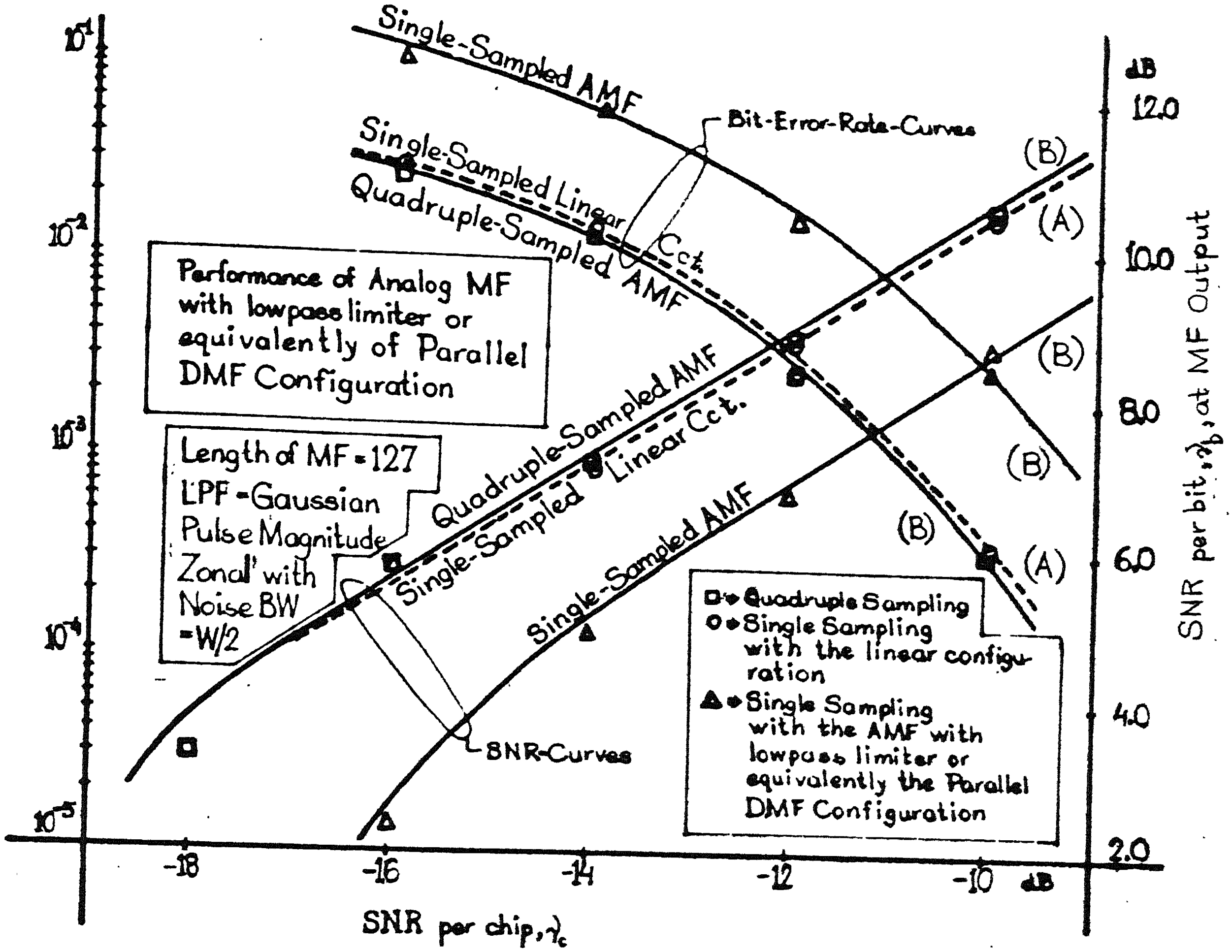

Figure 3.4 : Performance of Adaptive Digital Multisampling with Zonal Filters using the Analog Matched Filter for Spread Spectrum Systems.
(A) Single Sampled Linear Configuration,
(B) The Present Configuration.

the digital matched filter results in an extra 2 dB loss when multisampling is resorted to. For single sampling, however, the hard limiter associated with the DMF (digital matched filter) gets samples which are already hard limited by the lowpass limiter, and thus the inclusion of this extra hard limiter does not affect the output SNR. Therefore it is expected that the multisampled DMF can only approximate its own singly sampled version, or equivalently the singly sampled analog matched filter with lowpass limiter. The experimental results are shown in Fig. 3.5. It is thus easily understood that the extra 2 dB loss having been made indistinct, the single DMF circuit as proposed here will surely coperform the singly sampled linear circuit when multisampling is resorted to in the former. The next special DMF arrangement achieves this by allowing only digitized signals to enter the second hard limiter associated with the digital matched filters.

(C) Multiple Parallel Digital Matched Filters with Lowpass Limiter.

This special configuration shown in Fig. 3.6 is actually a digital equivalent realization of the analog matched filter with lowpass limiter. From the figure, it is seen that instead of adding the multisamples before matched filtering, the addition is done at the output of the matched filtering stage. The receiver processes each of the lowpass limited binary multisamples of a particular chip independently of the others and the sum of the optimally weighted outputs of the parallel DMFs is used as the decision variable. To elaborate, it suffices to mention that here there are N_s parallel independent DMFs, of which the k^{th} DMF ($k=1,2,\cdots,N_s$) processes the k^{th} multisamples of the N chips in the spread spectrum sequence of length N. The

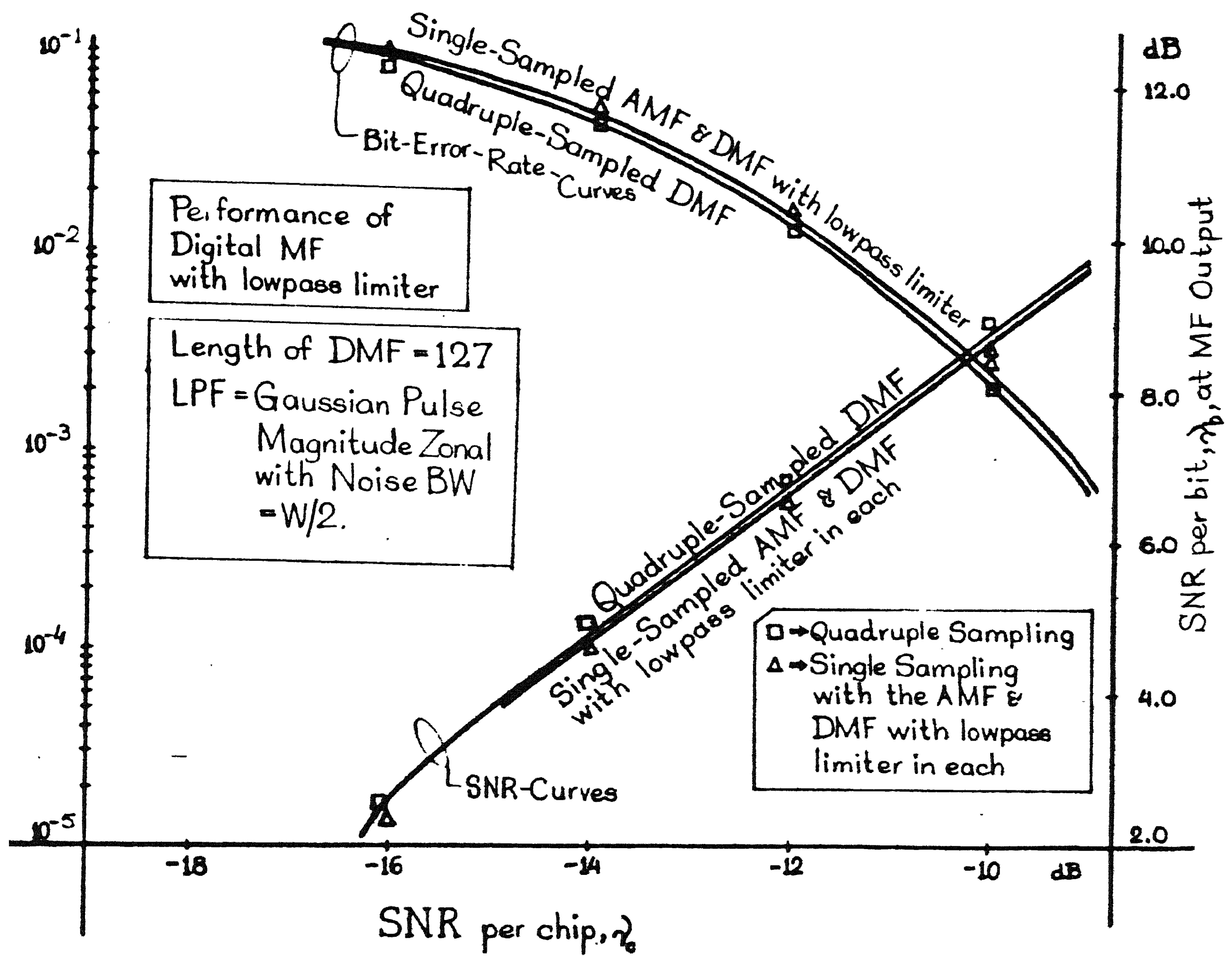

Figure 3.5 : Performance of Adaptive Digital Multisampling with Zonal Filters using the Digital Matched Filter for Spread Spectrum Systems.

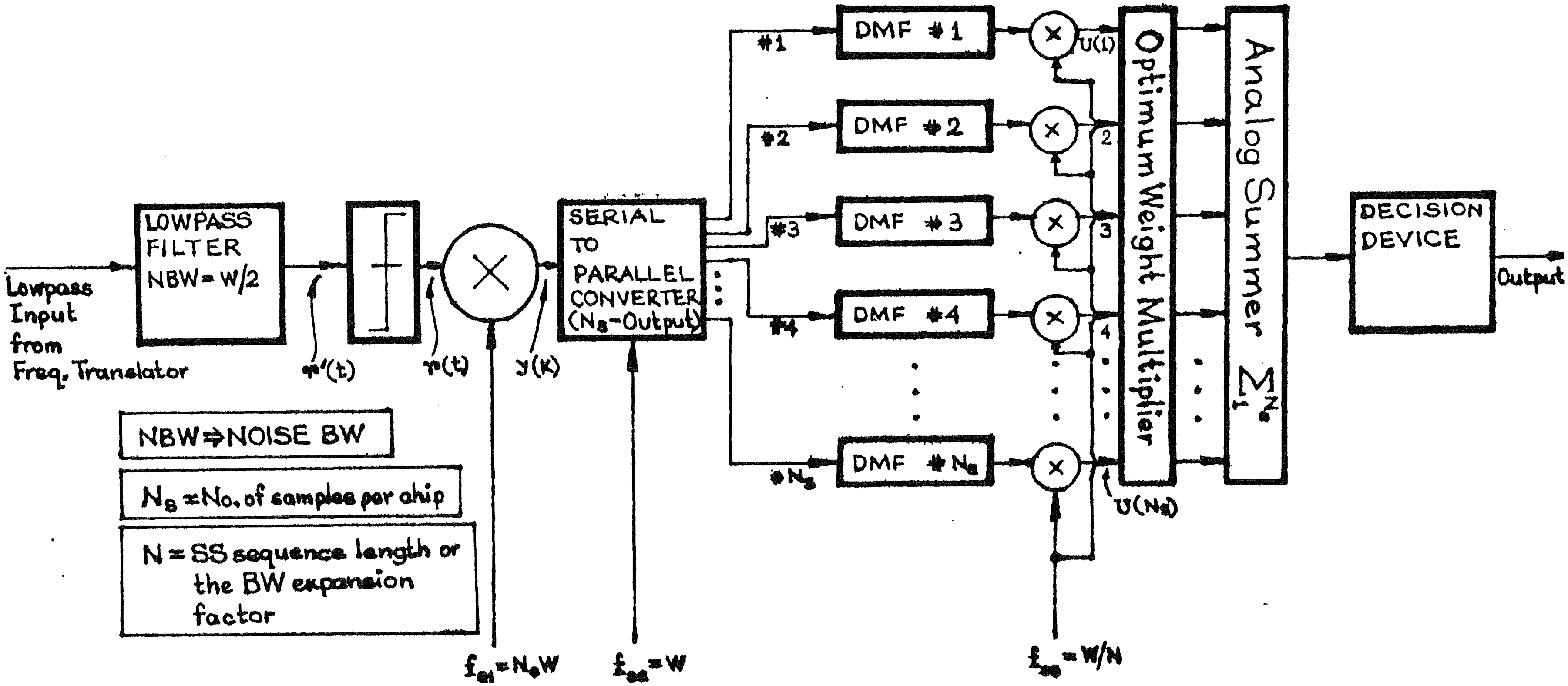

Figure 3.6 : Adaptive Digital Multisampling with the Multiple Parallel Digital Matched Filter for Spread Spectrum Systems.

claim that the present configuration is equivalent to the ahalog matched filter with lowpass limiter is substantiated by the arguments that follow.

Let the samples processed by the k^{th} DMF be denoted by $x(k,i)$, $i=1,2,\cdots,N$, $k=1,2,\cdots,N_s$, where, N is the length of the spread spectrum sequence. Let the stored replica be denoted by, $s_R(i)$, $i=1,2,\cdots,N$. The output of the k^{th} DMF is denoted by $U(k)$, $k=1,2,\cdots,N_s$, and is given by,

$$U(k) = \sum_{i=1}^{N} x(k,i) \cdot s_R(i) \quad . \qquad (3.2.11)$$

From Eq. (3.2.7), the covariance of $x(m,j)$ and $x(n,i)$ are readily determined to be,

$$Cov[x(m,j),x(n,i)] = A_c^{\,2} \cdot \exp\left[-1{\cdot}155\left|\frac{N_s(i-j)\cancel{+n-m}}{N_s}\right|\right] , \qquad (3.2.12)$$

where, the definition, $W=1/T$ has been used. The autocovariance function for $U(k)$, $k=1,2,\cdots,N_s$, may be given now by,

$$\begin{aligned} Cov[U(m),U(n)] = {} & Cov[x(m,1),x(n,1)] \cdot \sum_{i=1}^{N} s_R^{\,2}(i) \\ & +[Cov[x(m,1),x(n,2)]+Cov[x(m,2),x(n,1)]] \\ & \cdot \sum_{i=1}^{N-1} s_R(i) \cdot s(i+1) \\ & +[Cov[x(m,1),x(n,3)]+Cov[x(m,3),x(n,1)]] \\ & \cdot \sum_{i=1}^{N-2} s_R(i) \cdot s_R(i+2) \quad + \cdots \\ & +[Cov[x(m,1),x(n,N)]+Cov[x(m,N),x(n,1)]] \\ & \cdot s(1) \cdot s(N) \end{aligned}$$

$$\text{or,}\quad \mathrm{Cov}[U(m),U(n)] = \sum_{i=1}^{N} s_R^{\,2}(i) \cdot \mathrm{Cov}[x(m,1),x(n,1)] + \sum_{i=1}^{N-1} \sum_{j=1}^{N-i} s_R(j) \cdot s_R(j+i) \cdot [\mathrm{Cov}[x(m,1),x(n,i+1)]+\mathrm{Cov}[x(m,i+1), x(n,1)]] \qquad (3.2.13)$$

Since the spread spectrum sequence chips are generated by antipodal(+ve and -ve) pulses of equal amplitudes such that over a large length, the number of +ve chips and -ve chips are equal and the chips are uncorrelated from each other, the sum of the squares of the chip values will be much larger compared with those of the cross-terms. Quantatively, the sum of the squares will be exactly $NA_c^{\,2}$, while that of a cross-term will equal the values A_c or $-A_c$ each with probability 1/2 when the length N is odd, and the value zero with probability 1, when N is even. The 2nd, 3rd, ... covariance terms in Eq. (3.2.13) are less in magnitude than the covariance term in the first summation, as may be easily appreciated from Eq. (3.2.12), and therefore, the expression on the right hand side of Eq. (3.2.13) simplifies to,

$$\mathrm{Cov}[U(m),U(n)] \doteq NA_c^{\,2} \cdot \mathrm{Cov}[x(m,1),x(n,1)] , \qquad (3.2.14)$$

from which it is readily appreciated that the autocovariance function of the output U(k) is the same as, save a constant factor, that of x(k,i), for any $i=1,2,\cdots,N$. For any particular chip i, one can identify x(k,i) as the sample y(k) out of the multisampler. Consequently the performance of the present configuration is identical to that of the analog matched filter with lowpass limiter; and besides these the weights

in the analog matched filter with lowpass limiter are the same as the corresponding ones in the present configuration. The performance figures of the present configuration is then the same as that of the analog variety and is already shown in Fig. 3.4. Since a soft quantization usually calls for eight levels of quantization for an acceptable performance, the present circuit with quadruple sampling will indeed still be a considerable simplification.

3.3 Miscellaneous Experiments:

3.3.1 Performance of Adaptive Digital Multisampling with Nonzonal Filters:

In the previous section the performance of AMx has been analysed and experimented with using different receiver configurations with the basic assumption of that of a zonal filter. It has been mentioned before that digital multisampling is less affected by interchip interference as compared with analog multisampling. The present section tests the performance of digital AMx with interchip interference. The filters in the bandpass and lowpass stages are assumed to have a rectangular frequency response characteristic with linear phase and with noise bandwidths respectively of W and W/2. Such choices give rise to a good deal of interchip interference. The experimentation has been carried out with several sets of symmetric weighting factors. By symmetric weighting is meant that type of weighting in which the weights display a symmetry with respect to the centre of the chip. In this respect, all the weight-sets obtained so far, are symmetric. For quadruple sampling, the variation in the output SNR gain over single sampling is shown in Fig. 3.7, against that in the β-factor, defined as the ratio of the middle weights to

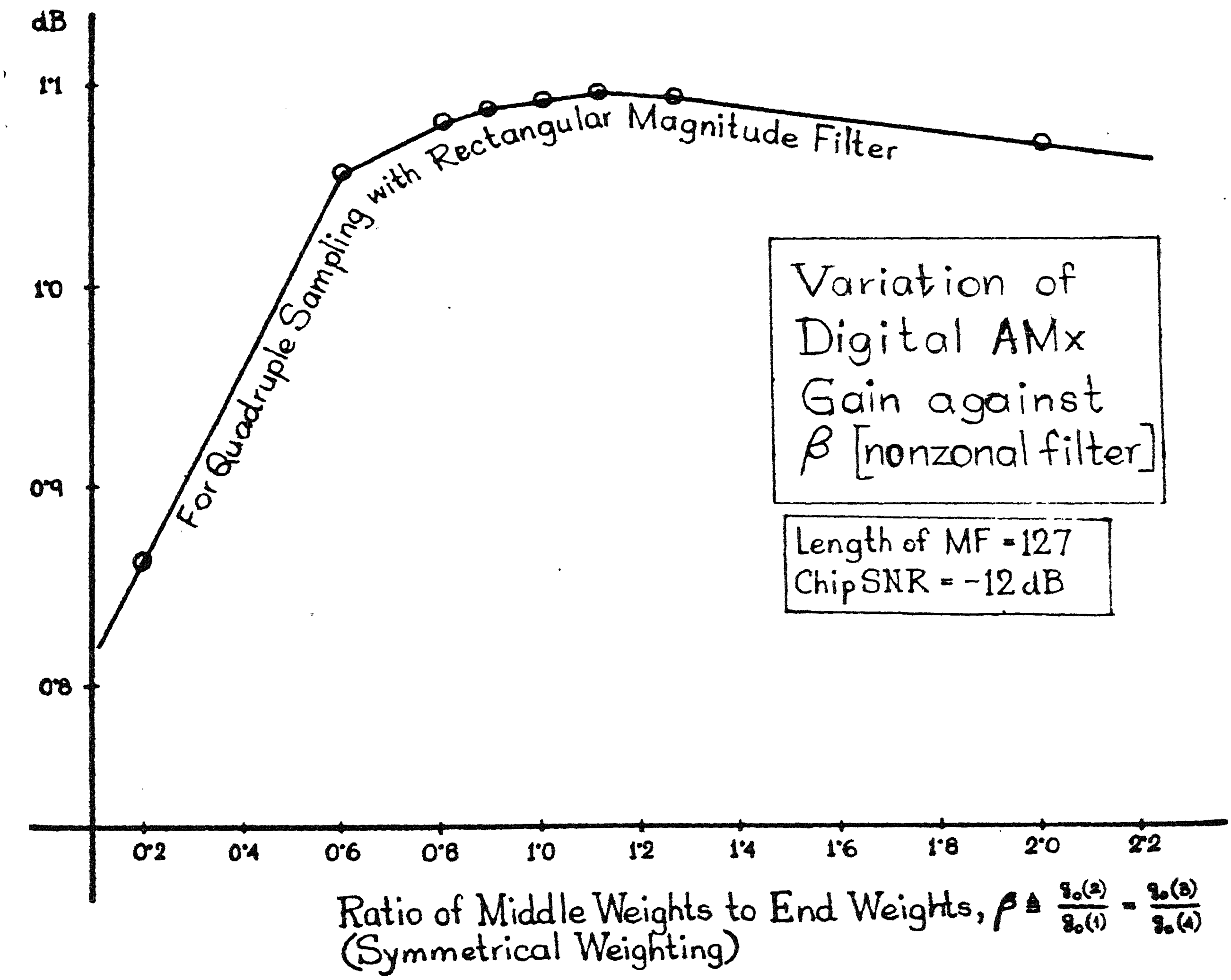

Figure 3.7 : Performance Variation of Adaptive Digital Multisampling with the β-factor for Nonzonal Filters and Very Low Input SNR.

the end weights,.

$$\beta = g_o(2)/g_o(1) = g_o(3)/g_o(4) \quad . \qquad (3.3.1)$$

It is observed from Fig. 3.7 that the variation in the SNR gain is small over a wide range of chosen β-factors. However, the optimal value of β appears to be somewhat near 1·1. This justifies the use of NAMx (for which β=1) by Das and Shanmugavel in their analyses of digital multisampling. Further, it also proves the conjecture (section 2.4) that digital multisampling is less affected by interchip interference; it is recalled that for NAMx, a perfect zonality would yield an SNR advantage of about 1·4 dB (section 2.3) referred to single sampling, while in the present case that gain is in the region of about 1 dB, thus showing only a 0·4 dB degradation in the performance of NAMx from zonal to nonzonal filters.

3.3.2 Performance Variation of Analog and Digital Multisampling against Variation in the Noise Bandwidth of the Filters Assumed Zonal:

If one increases the noise bandwidths of the filters in the receiver, then the autocovariance of the noise is reduced, and it is expected that the performance of multisampling should be better. Such is indeed the case as shown in Fig. 3.8, which plots the SNR gain due to AMx over single sampling as a function of the equivalent lowpass noise bandwidth. The optimum weights for each point plotted in the simulation figures have been calculated in the same manner as before. While the improvement in the performance of AMx may be apparently alluring, its practical values are not so when the noise from the channel at the RF front end has a wideband characteristic, as in the additive white noise channel. As the noise bandwidth is increased, more noise

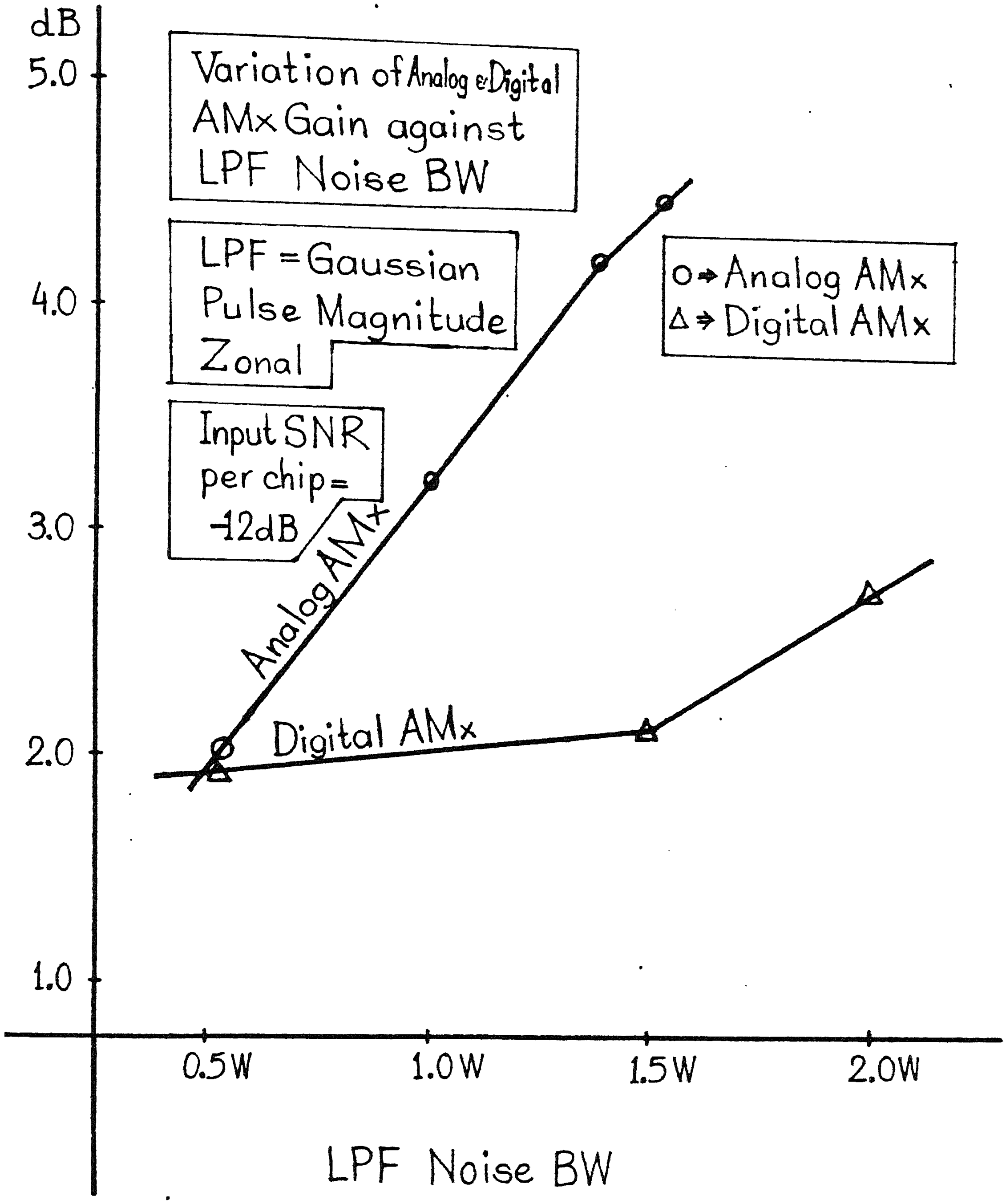

Figure 3.8 : Performance Variation of Adaptive Multisampling with the Noise Bandwidth of the Zonal Equivalent Lowpass Front End Filter.

power from the channel enters the receiver. It can be easily verified that the overall gain in SNR with a larger noise bandwidth is less compared with that with the standardized equivalent lowpass bandwidth of W/2.

However the principal advantage of increasing the bandwidth lies in an easier implementation of the criterion of zonality, although the channel bandwidth is inefficiently utilized. In multiaccess spread spectrum systems, it is usually the cochannel interference that dominates the additive noise. The relatively narrower bandwidth of the cochannel interference implies that increasing the bandwidth in such cases will not increase the noise power intercepted by the receiver significantly. Thus with multiaccess type of environment, multisampling with a larger noise bandwidth should be better compared with that with a smaller noise bandwidth.

3.4 Summary and Discussion:

This chapter has reported the theoretical as well as experimental results of adaptive multisampling for low rate coded communication systems, by concentrating on large length spread spectrum systems. This was preceeded by an appropriately elaborate discussion on the generalized principles and performance of adaptive multisampling. In all the experimental runs, a hard limiting of any low SNR analog signal has been seen to result in a 2 dB loss in the output SNR, which is exactly what was predicted earlier (section 2.2) on purely theoretical grounds. The loss, on the other hand, has been seen to have been adequately compensated for by the use of adaptive multisampling in several receiver systems. Performance of digital multisampling has also been examined

along with nonzonal filters having a rectangular frequency response with linear phase and has been seen not to differ from that with zonal filter by a large amount. For such interchip interference, the SNR gain due to digital multisampling over single sampling has been experimentally observed to be in the range of about 1 dB. In the last section, the performance variation of analog and digital multisampling has been studied with variation in the receiver noise bandwidth. It has been observed that the multisampler performs better and better as the noise bandwidth is increased.

To end this chapter, a final comment is now made regarding what actually adaptive multisampling refers to. For correlated noise, it is well known that the optimum way of processing a noisy signal is first to whiten the noise by means of a prewhitening filter and then perform a matched filtering on the output of the prewhitening filter. It has been discussed that adaptive multisampling is the optimum way of processing the noisy chips when the coefficient matrix G_o is given by Eq. (3.1.11). It should therefore be reasoned that as far as the processing of the chips is concerned, adaptive multisampling is equivalent to the prewhitening-cum-matched-filtering approach, more commonly known as the Wiener filtering. The demonstration is not very difficult. Any positive definite autocovariance matrix can be broken up as,

$$R = QQ^T \quad .$$

The whitening filter for this type of autocovariance matrix is easily represented as the matrix operator Q^{-1}, and thus its output has a signal vector, $Q^{-1}S$ mixed with white noise. The matched filter in the chain is then the vector operator $(Q^{-1}S)^T$. Thus the resulting output signal vector of the Wiener filter is $(Q^{-1}S)^T Q^{-1}S$, which simplifies to $S^T R^{-1}S$, showing that the Wiener filter is equivalent to the vector operator $R^{-1}S$, which is nothing but the coefficient matrix in Eq. (3.1.11).

CHAPTER 4: PERFORMANCE OF ADAPTIVE MULTISAMPLING IN HIGHER RATE CODED COMMUNICATION SYSTEMS

IN chapter 3, applications of adaptive multisampling in low rate coded communication systems were discussed by focusing attention on large length spread spectrum communication systems. There are common popular sytems that use much higher code rates. Common examples are systems without any coding or those with error correcting codes. The receiver front end (RF) is linear for such cases as mentioned in section 1.1. The present chapter examines the performance of this type of communication systems in the light of multisampling. First, hard decision decoders of convolutional codes are considered with multisampling to see whether multisampling can extract soft decision performance out of these decoders. This is done with the assumption of a zonal filter[1]. Next, digital multisampling is taken up for a receiver configuration using multiple parallel decoders similar to the multiple parallel DMF discussed in section 3.2. This is also based on the assumption of a zonal filter[1]. The next section discusses nonzonal filters and the performance of adaptive analog multisampling with nonzonal filters.

---

[1]It is to be remembered that for high SNRs, the sample error probabilities are strong functions of the interchip interference. This is why, the requirement of zonality in the case of high rate coded communications is much more stringent than that in the case of lower rate coded communications. Nevertheless, the performance of multisampling with the assumption of zonality is experimented with in order to obtain a measure of the merits of multisampling.

4.1 Performance of Hard Decision Decoders with Adaptive Multisampling and Zonal Filters:

4.1.1 Adaptive Analog Multisampling:

As in section 3.2.1, the lowpass filter is assumed to stand for the equivalent lowpass representation of the cascade of the linear bandpass stage with the lowpass stage through the frequency translator. This equivalent low-pass filter is as before assumed to have a Gaussian frequency response magnitude function with linear phase, and having a 3 dB cutoff frequency of W/2. The SNR gain due to analog AMx over single sampling has already been calculated in section 3.2.1(A), and are reproduced here for quick reference.

$$\Delta\, SNR_{1x}^{AMx} \doteq 2{\cdot}45 \text{ dB} , \tag{4.1.1}$$

$$\Delta\, SNR_{1x}^{NAMx} \doteq 1{\cdot}74 \text{ dB} , \tag{4.1.2}$$

and the weighting factors are given by,

$$\begin{aligned} g_o(1) &= g_o(4) = 1{\cdot}000 , \\ g_o(2) &= g_o(3) = -0{\cdot}091 . \end{aligned} \tag{4.1.3}$$

These gains therefore reflect in the performance of the decoders as verified experimentally using hard decision syndrome decoding algorithm of Massey for systematic convolutional codes and hard decision Viterbi decoding algorithm for nonsystematic codes. The results are shown in Figs. 4.1 (a) and (b) respectively. In Fig. 4.1(b) the soft decision upper bounds are also shown for demonstrating that the multisampled hard decision decoders can outperform their soft

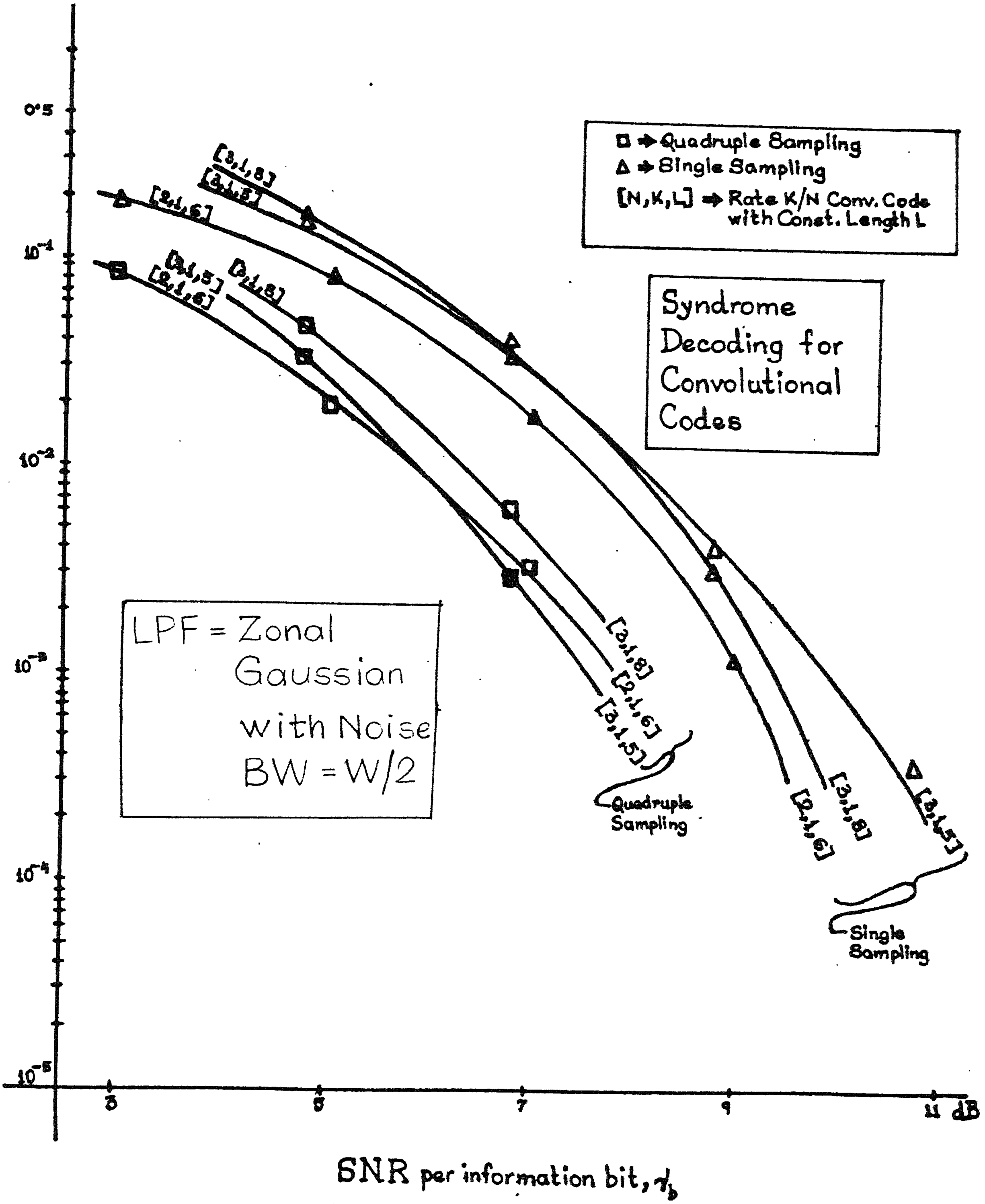

Figure 4.1(a) : Performance of Adaptive Analog Multisampling with Zonal Filters for Hard Decision Syndrome Decoding Algorithm (Systematic Convolutionally Coded Data).

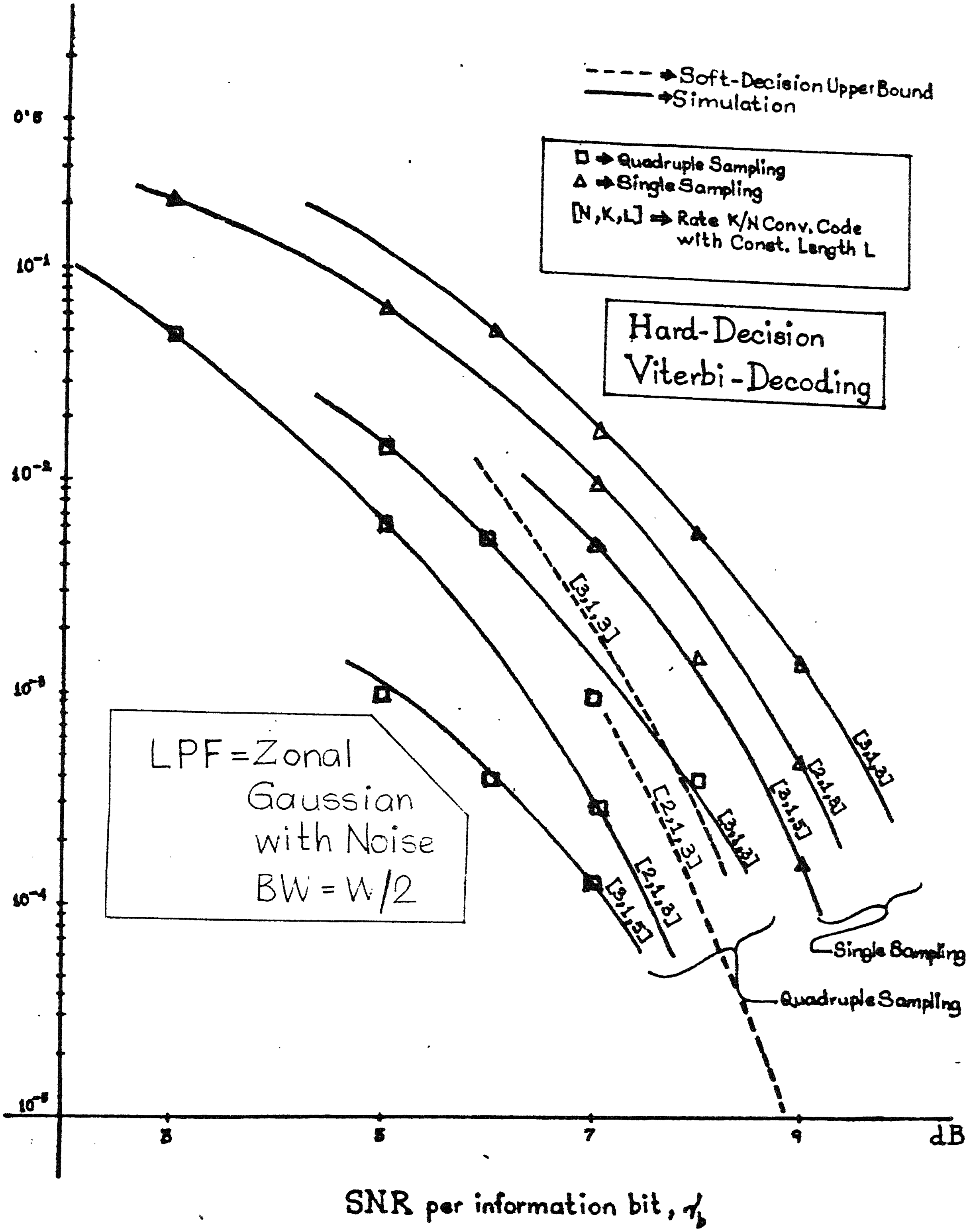

Figure 4.1(b) : Performance of Adaptive Analog Multisampling with Zonal Filters for Hard Decision Viterbi Decoding Algorithm (Nonsystematic Convolutionally Coded Data).

decision single sampled counterparts, provided that the filters in the receiver are zonal.

4.1.2 Adaptive Digital Multisampling - the Multiple Parallel Convolutional Decoder Configuration:

Just as in the parallel DMF configuration of section 3.2.2(C), there is a 'dedicated' decoder for each set of corresponding samples of all the received code chips. The corresponding decoded bits from from the parallel decoders are weighted and added to form the decision variables for the detection of the corresponding bits. The weights are rather difficult to find out analytically and a trial-and-error method is to be used to find out the optimum weights. In the experiment, Viterbi decoding (hard decision) has been examined with a rate 1/3 nonsystematic code, having constraint length of 3. The optimum weights have been found out to be symmetrical with a β-factor around unity, indicating that in such cases NAMx would be nearly as good as AMx. The SNR gain over single sampling is however rather small and is about 1 dB, compared with the 2 dB plus gain of the much simpler single decoder configuration of the last subsection. This is why the present configuration is of little practical value. Fig. 4.2 shows the performance curves.

4.2 Miscellaneous Experiment - Performance of Adaptive Analog Multisampling with Nonzonal Filters:

The purpose of this section is to investigate the performance of analog AMx when there is considerable interchip interference. The nonzonal lowpass equivalent filter is assumed to have a rectangular frequency response magnitude characteristic with linear phase and cutoff frequency

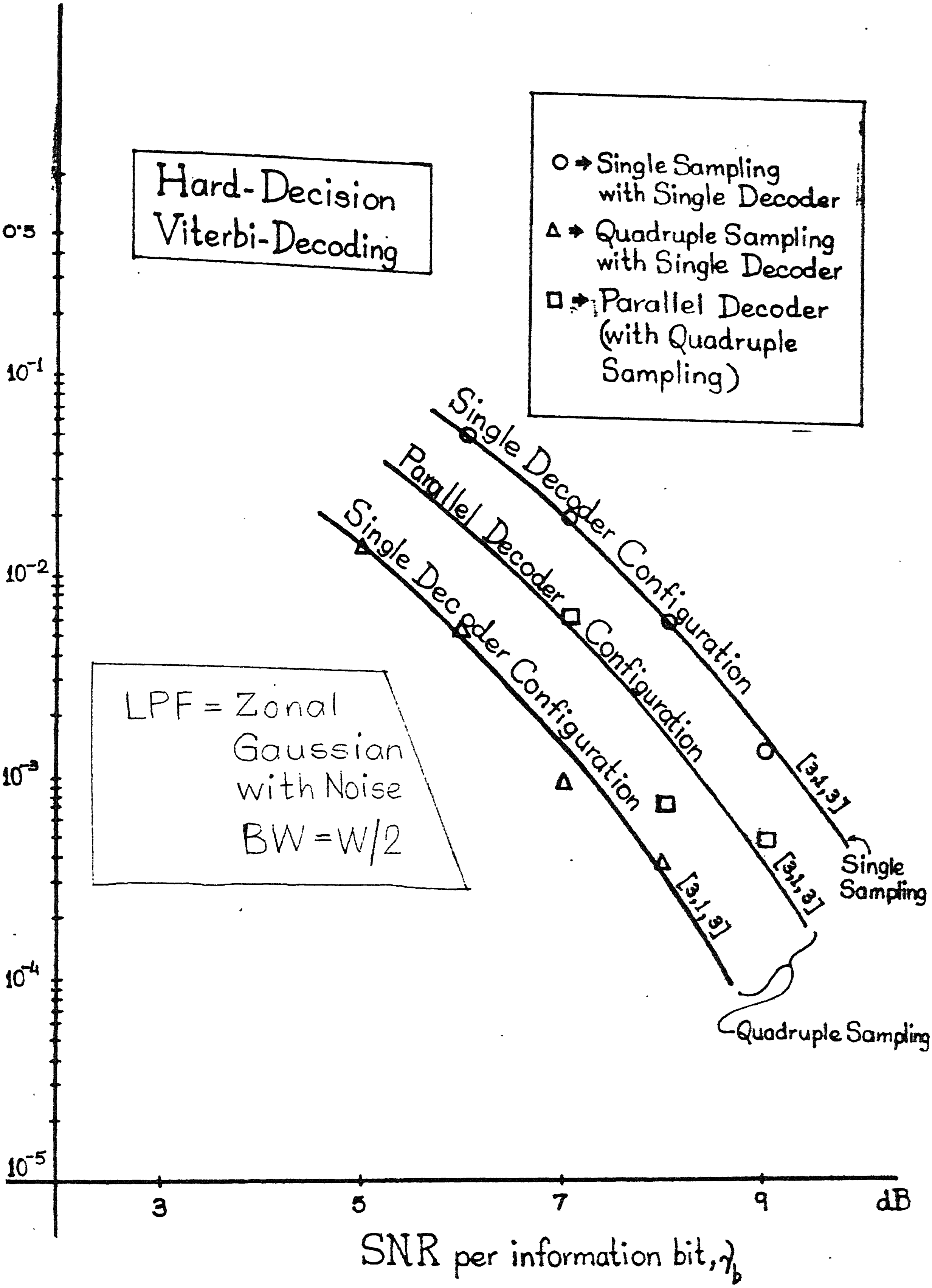

Figure 4.2 : Performance of Adaptive Digital Multisampling with Zonal Filters using the Multiple Parallel Hard Decision Viterbi Decoder Configuration.

of W/2 as before. For this type of a filter, or for that matter for any commonly used filter, the dominant contribution to the nonzonal interchip interference in any chip comes mainly from its immediate predecessor in the case of causal filters and from its immediate successor in addition to the previous one in the case where the filters are non-causal. The choice of the filter transfer function in the present analysis has been a noncausal one and so one must take into account the states of the last and the next chips in order to determine the effect of interchip interference in a present chip. Thus there can be at most four distinct types of interchip interference on any particular chip, depending on its state, the state of its predecessor and that of its successor, as indicated in the following table.

TABLE 4.1

Serial No.	Predecessor	Present	Successor
1	1/0	1/0	1/0
2	0/1	1/0	0/1
3	0/1	1/0	1/0
4	1/0	1/0	0/1

Now for the rectangular magnitude filter, the autocovariance of the noise at the filter output is given by ,

$$\varphi_r(\tau) = \varphi_r(0) \cdot \frac{\mathrm{Sin}[2\pi f_c \tau]}{2\pi f_c \tau} , \qquad (4.2.1)$$

where f_c is the cutoff frequency and is equal to W/2 for the rectangular filter to have a noise bandwidth of W/2. For quadruple sampling, the noise autocovariance matrix becomes,

$$R = \varphi(0) \begin{bmatrix} 1\cdot000 & 0\cdot900 & 0\cdot637 & 0\cdot300 \\ 0\cdot900 & 1\cdot000 & 0\cdot900 & 0\cdot637 \\ 0\cdot637 & 0\cdot900 & 1\cdot000 & 0\cdot900 \\ 0\cdot300 & 0\cdot637 & 0\cdot900 & 1\cdot000 \end{bmatrix} , \qquad (4.2.2)$$

and the inverse of this matrix is given by,

$$R^{-1} = \frac{1}{\varphi(0)} \cdot \begin{bmatrix} 68\cdot831 & -164\cdot988 & 150\cdot744 & -51\cdot219 \\ -164\cdot988 & 426\cdot200 & -414\cdot154 & 150\cdot744 \\ 150\cdot744 & -414\cdot154 & 426\cdot200 & -164\cdot988 \\ -51\cdot219 & 150\cdot744 & -164\cdot988 & 68\cdot831 \end{bmatrix} \qquad (4.2.3)$$

The signal components of the samples under the four different chip sequences of interest alongwith the corresponding gain over single sampling and the optimum weighting factors can now be easily calculated[2]. The calculated results are shown summarily shown in Table 4.2. In this table, all the signal components shown have been normalized with respect to A_c, i.e., the undistorted chip magnitude while all the weights shown have been normalized with respect to the standard deviation of the noise samples. As in section 3.1, $s(k_o)$ denotes the signal component in the case of single sampling (optimum pt.).

[2]Calculation is done using Eqs. (3.1.12),(3.1.15) with (4.2.3), and the signal components themselves (calculated)in order to find out the gain and the weighting factors tabulated.

Sl. No.	Chip Sequence	Signal magnitudes normalized with respect to Ac					Normalized Optimum Weighting Factors				AMx Gain over Single Sampling in dB
		$\lvert s(1)\rvert$	$\lvert s(2)\rvert$	$\lvert s(3)\rvert$	$\lvert s(4)\rvert$	$\lvert s(k_o)\rvert$	$g_o(1)$	$g_o(2)$	$g_o(3)$	$g_o(4)$	
1	111/000	1.000	1.000	1.000	1.000	1.000	2.205	-1.440	-1.440	2.205	3.69
2	010/101	0.409	0.704	0.704	0.409	0.744	-2.367	2.225	2.225	-2.367	4.11
3	011/100	0.248	0.694	1.010	1.161	0.872	-3.603	8.984	-8.634	4.025	3.43
4	110/001	1.161	1.010	0.694	0.248	0.872	4.025	-8.634	8.984	-3.603	3.43

TABLE 4.2 : Pertaining to the Rectangular Magnitude Filter Having a Linear Phase Response Characteristic and Adaptive Analog Multisampling with this Filter

For the particular noncausal filter that is being considered, the single sampling is done at the centre of the respective chip interval.

Although the optimum SNR gains are seen to be fairly encouraging (the minimum being 3·43 dB), the actual implementation of these is far more complicated than expected. One has to know beforehand the present chip as well as its immediate successor in order to calculate the amount of inter-chip interference on the present chip. This is ironical. It can be easily verified that when there is a mismatch between the set of weights and the nature of interchip interference prevailing in the chip, the process of AMx results often in an SNR loss rather than giving any gain. The most logical way to get rid of this puzzle is then to employ an algorithm that is somewhat similar in principle to the optimum M-ary detection of signals in presence of noise. In the present context, the M signals are the different (altogether four in number) shapes of the chips with interchip interference. All the four different sets of weights are to be used with each chip to form decision variables, and then the maximum out of these is selected and detected. As the signalling is antipodal, one has to form two decision variables for each type of interference, which would be equal in magnitude but have different signs. Thus there would be eight decision variables for each chip to choose from and the largest out of these is to be found out. Once this is done, one can trace back to find out the chip polarity to which the weights generating this decision variable are matched. The polarity would give the required state of the present chip.

However such a 'Choose-the-Largest' type of algorithm fails to show any marked improvement over single sampling, owing to the fact that in this algorithm, it is

precisely half the distance between the largest positive and the largest negative decision variables that serves as the measure of the maximum amount of noise that can be tolerated before an error is committed in the detection of the chip. In single sampling, on the other hand, the signal component of the single sample is itself the measure. Now, it can be easily verified that this measure in the algorithm just described is always less than that in single sampling for the four different types of interchip interference. This is why single sampling proves better, as demonstrated in the experimental results in Fig. 4.3.

However, as stated earlier, analog AMx in the case with interchip interference will be of practical value if the chip interference is known beforehand. The irony of this requirement in high rate coded systems has already been pointed out. But in lower code rates, if a repititive type of coding with or without polarity modulation is used, just as in spread spectrum systems, then the polarity modulation having been known, the predecessor and successor of each chip (save the two extreme end chips in the sequence) are also known, and thus the interchip interference in the chip being processed is also very accurately known. Thus an AMx to detect the intermediate chips in the sequence (now with truly optimum weighting) and single sampling to detect the end chips may constitute an effective detection algorithm for the detection of the chips, and for sequence lengths in excess of about 10, there will be an overall gain in the SNR per information bit of about 3·5 dB, compared to single sampling used to detect all the chips.

The simulation of this algorithm has not been carried out owing to CPU time limitations. Instead, what truly optimum weighting can provide has been examined

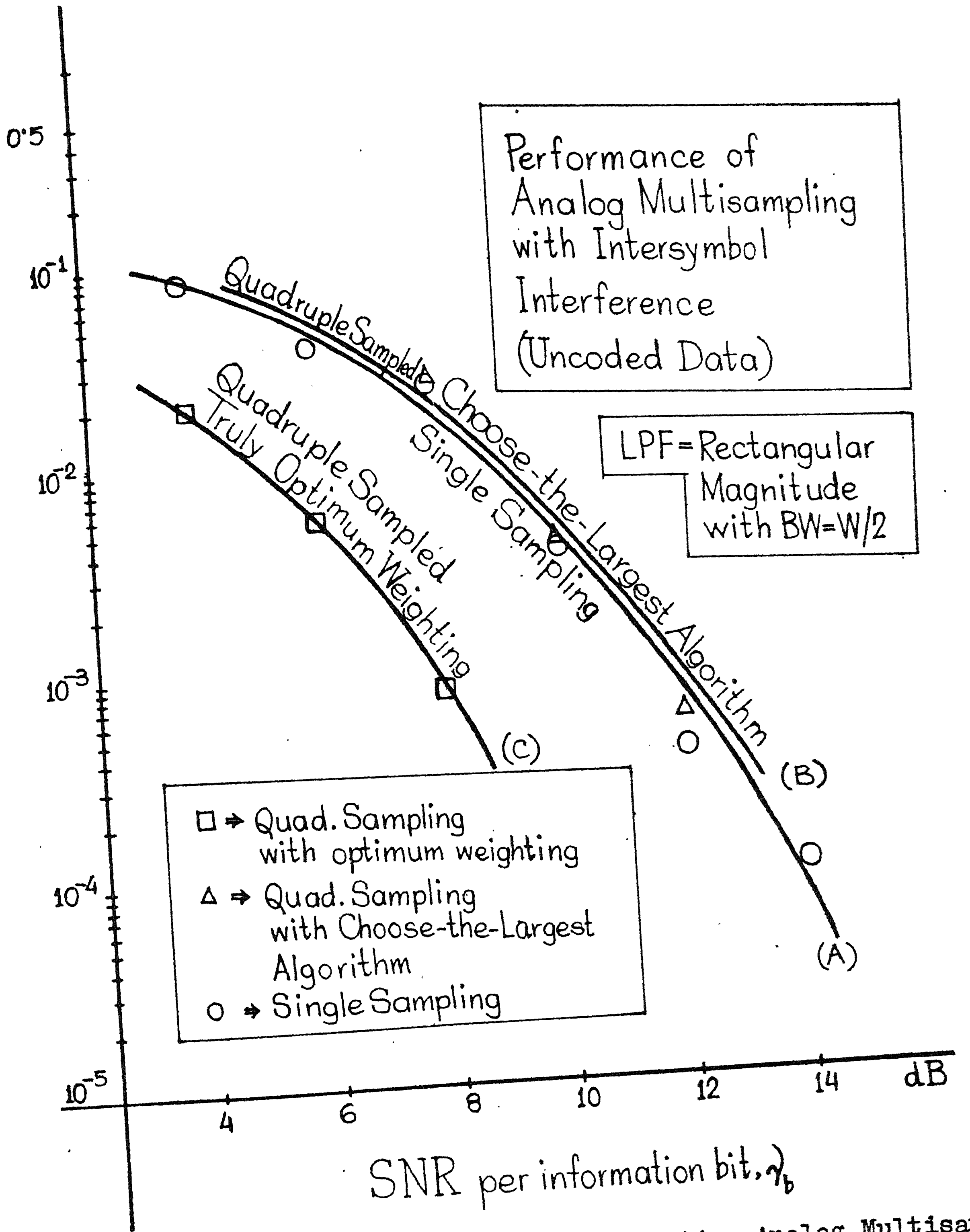

Figure 4.3 : Performance of Adaptive Analog Multisampling for Uncoded Data having Intersymbol Interference (A) Single Sampling, (B) Quadruple Sampling with Choose-the-Largest Algorithm, (C) Quadruple Sampling with Truly Matched Optimum Weighting.

using uncoded data. The results are shown in Fig. 4.3. The experimental curves indeed display the near 4 dB advantage due to analog AMx over single sampling when the weights are optimum.

4.3 Summary and Discussion:

In this chapter first analog multisampling has been considered with zonal filters using syndrome as well as Viterbi decoding algorithms for decoding systematic and nonsystematic convolutional codes. It has been seen that with analog AMx and zonal filtering hard decision decoders can indeed coperform their soft decision counterparts. Next, digital multisampling with zonal filter has been taken up using a parallel Viterbi decoder configuration with post-decoder weighted addition. This however shows a smaller gain (of about 1 dB) over single sampling although the configuration is much more complex than the previous ones. The next section has discussed some aspects of analog AMx with severe interchip interference. A method for obtaining the calculated SNR gains with matched weighting has been suggested for low rate repititively coded communication systems. It has been experimentally verified that with matched (optimum) weighting, analog AMx can provide a near 4 dB SNR gain compared to the conventional single sampling.

CHAPTER 5: METHODS FOR THE SIMULATION

THE previous chapters have dealt in detail with the analyses of the results obtained theoretically and the validation thereof by simulation on a digital computer. This chapter briefly reports the methods used in the simulation.

5.1 The Basic Components of the Simulation Algorithm:

The simulation algorithm can be broken up into several major sections like the following.

(a) Generation of data bits and the encoding of these data into coded form by means of a prescribed coding algorithm;

(b) Generation of random noise numbers;

(c) Convolution of the noise numbers with the impulse response of the lowpass filter;

(d) Convolution of the code chips with the impulse response of the lowpass filter;

(e) Obtaining multiple samples of each of the two convolution operations;

(f) Addition of the multiple samples of the code chips with those of the convolved noise;

(g) Weighted addition of the multiple samples for each chip interval to form the decision variable;

(h) Detection of the chips and decoding; and,

(i) Calculation of bit SNR and bit-error-rate.

The above steps are executed repeatedly, each time with a fixed and conveniently small number of data bits. In this way, effectively a large number of data bits gets examined without increasing the memory requirement irreasonably. In the actual simulator software, the above steps have been implemented as described below.

5.2 Implementation of the Basic Steps:

(a) Generation of Data Bits and their Encoding.

The data bits are generated by hard clipping of the output of a uniformly distributed random number generator. The clipping threshold is placed at the mean or average value of the generator. Whenever the random number generated is found greater than **this**,the corresponding bit is declared to be positive (i.e., a 1). Otherwise, the bit is declared negative (i.e., a 0). This type of data bit generation has all the important properties that a true pseudonoise sequence is required to have. Owing to the uniform distribution of the generated numbers around the fixed clipping threshold, the number of positive and negative bits over a large number of bits are equal, as also are the no. of n-tuples of positive bits and that of negative bits, for $n=2,3,\cdots$. But for this the uniform deviates must be nearly independent of each other. After the bits are generated,the encoding is carried out using a prescribed coding scheme under use.

(b) Generation of Random Noise Numbers.

Random numbers with Gaussian distribution can be generated using standard software if available. If not available, the uniform generator used earlier can once again be used with the transformation,

$$n = [\sqrt{(-2\log_e r_1)}] \cdot \mathrm{Cos}(2\pi r_2) \quad , \tag{5.2.1}$$

where, r_1 and r_2 are two uniformly distributed random numbers each with mean 0·5 and unit variance. The normal deviate **n** so generated has zero mean and unit variance. This has to be multiplied by the desired standard deviation in order to get the desired variance for the normal deviates.

(c), (d), (e)

Convolution (Filtering) and Multisampling.

While the order of the steps given above relates more to the actual system implementation, the simulation is done in a slightly different order and manner. First, in order to convolve, samples are to be generated of both of the time functions being convolved. For the noise numbers, the numbers are themselves used as the samples, and are therefore generated at the sampling rate necessary for convolution. This rate is precisely either the one at which the multisampling is carried out or the one of eight times per chip depending on which one is larger. The minimum allowed rate is thus eight times per chip which has been arrived at through the experimental observations with known results. The convolution of the random numbers is then performed using the discrete convolution defined earlier in chapter 3. The convolved output is appropriately normalized so that the output convolved noise numbers also have the desired variance. The output is taken at the appropriate sampling instants.

When it comes to the convolution of the chips (which is only necessary in the case of nonzonal filtering), it is easier to analytically calculate the sample values at the appropriate sampling instants and multiply the multisamples of the 'flat top' generated chips by these coefficients. For

the case of zonal filters, the multisamples of the generated chips can directly be used without any coeffient multipliers.

(f) and (g)

Mixing of the Noise with Signal and Weighted Addition.

These are quite straightforward once the multisamples of both the chips and the convolved noise are obtained. The corresponding multisamples of the code chips and the convolved noise are then added and appropriately weighted. After weighting, the noisy multisamples of each chip are separately added to form the decision variables for the chips.

(h) Detection and Decoding.

The detection refers to the process of making the chips fit to enter the decoding circuit that follows. This means that for decoders like the analog matched filter or any soft decision decoder, the chip decision variables obtained earlier are themselves fed unmodified into the decoder, while for hard decision decoders like the digital matched filter or the hard decision decoders for convolutional considered in chapter 4, the detection consists in taking a hard decision on the decision variables. The decoding is then done using the prescribed decoding rule.

(i) Calculation of Bit SNR and Bit-Error-Rate.

For the output SNR the mean and the variance of the output bit of the decoders prior to detection are to be computed, over a large number of iterations. Using then an SNR equation similar to the ones described in chapter 3 one can obtain the output bit SNR. The method just described is a simple one but is limited only to the cases where the decoder output is inherently analog as in matched filters.

For the calculation of bit-error-rate, the simple Montė-Carlo technique [14] is used. Over a large number of detected data the number of disagreements with the original data are found out by comparison, and this number is then divided by the number of data bits tested (either original or the detected ones - but not both). The quotient so obtained gives the bit-error-rate. A modified Montė-Carlo technique using the method of importance sampling [17] can also be used if the experimental observations are required to yield an error-rate resolution of the order of 10^{-5} or even less. But as in the present thesis interest has only been focused on an error-rate resolution of 10^{-4}, the use of simple Montė-Carlo technique as described above is more attractive due to its simplicity.

CONCLUSION AND SUGGESTIONS FOR FURTHER WORK

IN the present thesis a technique of improving the SNR performance in conventional low and high rate coded communication systems has been discussed. Adaptive Multisampling, as the technique has been termed, has been seen to provide substantial gain in SNR over the conventional systems using single sampling per chip. The basic premise of all the calculations has been coherent communication. The requirement of chip synchronization has been understood to be more stringent in the cases of high rate coded communications and nonzonal filters. It is therefore concluded that the predicted simplification in the design of the receivers is possible only when tight synchronization between the receiver and the transmitter can be established, and more so when the filters involved in the receiver are zonally designed. A point to be taken care of in this context is that it will be rather unwise to try to replace multibit quantization wherever possible by multisampled systems owing to the problems of tight synchronization and zonal filters in the latter. It is better to even have a judicious coexistence of these two in coherent systems such that the overall efficiency is really cost-effective.

The thesis has examined only random Gaussian noise as the dominating perturbation. Further the scope of the analysis of nonzonal filtering has been rather restricted. Further work may be undertaken on these aspects of Adaptive Multisampling. For example, under jamming environment or dominant cochannel interference situations, trial-and-error weighting can be carried out with and without zonal filters. Analog multisampling could also be another point of investigation under nonzonal filtering.

APPENDIX A-1

Minimum SNR Necessary to Transmit Information at a Rate Equal to the Channel Capacity [9]:

The channel capacity of a Binary Symmetric Channel (BSC) is given by,

$$C_{BSC} = 1 + p\cdot\log_2 p + (1-p)\cdot\log_2(1-p) \quad , \qquad \text{(A1.1)}$$

where, p is the transition probability, given by Eqs. (2.2.1) and (2.2.3b) as,

$$p = \operatorname{erfc}[\sqrt{(\gamma_b R_c)}] \quad . \qquad \text{(A1.2)}$$

It is known that the capacity gives the maximum rate of information transmission [11] for a particular value of p. In knowing the minimum SNR per information bit necessary to transmit information at a rate R_c, one has to set $C=R_c$ in (A1.1) and then using (A1.1) and (A1.2) determine the value of γ_b for the particular value of R_c under use.

Now as the code rate approaches zero, one can use the asymptotic expression for erfc(x) given in Eq. (2.2.5) into Eq. (A1.2) and write,

$$p \doteq \tfrac{1}{2} - \sqrt{[\tfrac{1}{2}\pi\gamma_b R_c]} \quad . \qquad \text{(A1.3)}$$

Further for $|x| \ll 1$,

$$\log_2(1+x) = \frac{1}{\log_e 2} \cdot \log_e(1+x) \doteq x/\log_e 2 \quad .$$

(A1.4)

Using (A1.3) and (A1.4),

$$\log_2 p \doteq -\left[1 + \frac{1}{\log_e 2}\sqrt{(2\gamma_b R_c/\pi)} \right] ,$$

$$\text{and} \quad \log_2(1-p) \doteq -\left[1 - \frac{1}{\log_e 2}\sqrt{(2\gamma_b R_c/\pi)} \right] . \tag{A1.5}$$

Substituting (A1.3) and (A1.5) back into (A1.1),

$$C_{BSC} \doteq \frac{2\gamma_b R_c}{\pi \log_e 2} . \tag{A1.6}$$

Then setting $C_{BSC}=R_c$, one readily obtains,

$$\gamma_b = \gamma_b^H = \tfrac{1}{2}\pi \log_e 2 \doteq 0\cdot 37 \text{ dB} , \tag{A1.7}$$

and this is for low rate codes (R_c approaching zero).

Next, for soft decision on the same type of channel, the capacity in bits per code symbol is given by,

$$C_{SDD} = \tfrac{1}{2} \cdot \sum_{i=0}^{1} \int_{-\infty}^{\infty} p[y|i]\cdot \log_2(p[y|i]/p[y])\cdot dy, \tag{A1.8}$$

in which y denotes the analog (unquantized) value of the received signal, at the instant of sampling the lowpass filter output and i is the transmitted binary digit. In the case where the additive noise is white gaussian, the noise component of y is also Gaussian, i.e.,

$$p[y|i] = \frac{1}{\sqrt{(2\pi\sigma^2)}} \cdot \exp[-(y-m_i)^2/(2\sigma^2)] , \tag{A1.9}$$

where, $m_i=A_c$, if $i=1$, and $m_i=-A_c$, if $i=0$, and σ^2 is the variance of the noise component in y.

Now,

$$p[y|1]/p[y] = 2/[1+\exp(-2A_c y/\sigma^2)] \quad ,$$

$$\text{and} \quad p[y|0]/p[y] = 2/[1+\exp(\ 2A_c y/\sigma^2)] \quad . \tag{A1.10}$$

For the code rate R_c approaching zero,

$$2A_c y/\sigma^2 = \frac{2\cdot y}{\sigma} \cdot \sqrt{\gamma_b R_c} \longrightarrow 0 \quad ,$$

and one can approximate (A1.10) as,

$$p[y|1]/p[y] \doteq 1 + \frac{y}{\sigma} \cdot \sqrt{(\gamma_b R_c)} \quad ,$$

$$\text{and} \quad p[y|0]/p[y] \doteq 1 - \frac{y}{\sigma} \cdot \sqrt{(\gamma_b R_c)} \quad . \tag{A1.11}$$

Using now the approximation given in (A1.4), one can have

$$\log_2(p[y|1]/p[y]) \doteq \frac{y}{\sigma} \cdot \sqrt{(\gamma_b R_c)} \quad ,$$

$$\text{and} \quad \log_2(p[y|0]/p[y]) \doteq \frac{y}{\sigma} \cdot \sqrt{(\gamma_b R_c)} \quad . \tag{A1.12}$$

Using these approximations in Eq. (A1.8), it follows that,

$$C_{SDD} = \frac{\gamma_b R_c}{\log_e 2} \quad . \tag{A1.13}$$

Setting $C_{SDD}=R_c$,

$$\gamma_b = \gamma_b^S = \log_e 2 = -1\cdot 60 \text{ dB} \quad , \tag{A1.14}$$

which is true for low rate codes. For other code rates such simplifying assumptions are not possible and one must resort to a numerical solution technique to find out γ_b.

APPENDIX A-2

Relation between the Chip-Error-Rate and the Output SNR per Information Bit for Digital Matched Filtering of Large Length Spread Spectrum Sequences:

Let N_{chip} denote the number of chips **taken** by the hard quantizer at the input of the digital matched filter. Using the definition for the SNR of the hard limited chips,

$$\gamma_c^H = \frac{\text{Squared mean value of the hard quantizer output}}{\text{Variance of the hard quantizer output}},$$

one readily obtains,

$$\gamma_c^H \doteq \frac{(N_{chip} - 2 \cdot N_{error})^2}{N_{chip}^2 - (N_{chip} - 2 \cdot N_{error})^2}, \qquad \text{(A2.1)}$$

where, the basic assumptions have been, (i) the hard quantizer quantizes antipodally, and (ii) N_{chip} is very large, so that N_{error}, i.e., the number of chips out of the N_{chip} chips in error is very nearly proportional to N_{chip}.

Recognizing now that under assumption (ii), the chip error probability is simply,

$$p \doteq \frac{N_{error}}{N_{chip}}, \qquad \text{(A2.2)}$$

one can simplify Eq. (A2.1) to read,

$$\gamma_c^H \doteq \frac{(1-2p)^2}{4p(1-p)^2}. \qquad \text{(A2.3)}$$

Since the length of the spread spectrum sequence and hence that of the matched filter are large, the output SNR per information bit from the matched filter will be N times the input SNR per chip, where N is the length of the spread spectrum sequence. Therefore, the SNR output per information bit is given by,

$$\gamma_b^H \doteq N \cdot \frac{(1-2p)^2}{4p(1-p)^2} , \qquad (A2.4)$$

which is the required equation.

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